

1973

An analysis of tests and objectives of elementary school mathematics

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An analysis of tests and
objectives of elementary
school mathematics

by

William Fredrick Coulson

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
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INTRODUCTION

One of the major concerns of educators today is the measurement of academic growth or achievement in various subject matter areas. In order to do this with any degree of success, two factors must be considered, the objectives which are to be attained and the instrument or device used to measure the degree of attainment of these objectives.

During the past two decades, many changes have taken place in the field of education in general and in mathematics education in particular. Included among these changes are those involving learning theory, teacher education, development of new methods, materials, and technology, as well as changes in our society. All of these changes have influenced the objectives of education and, consequently, the means of evaluating those objectives. During this same period, mathematics education has undergone many changes. "Modern mathematics" has become a familiar phrase in educational circles. Very briefly, modern mathematics may be characterized as being more mathematically oriented than was the mathematics in the curriculum prior to 1950, for instance. Mathematics itself is studied for the sake of its own theory, not just for the sake of its practicality. Mathematical concepts are used to unify the development of knowledge. "Why," or the mathematical theory behind a particular notion, is studied as much, or more than the "how," or the performance of particular operations, of mathematics.

The world of today demands more mathematical knowledge on the part of more people than the world of yesterday and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; and understanding the role of mathematics in our society is now a prerequisite for intelligent citizenship. Since no one can predict with certainty his future profession, much less foretell which mathematical skills will be required in the future by a given profession, it is important that mathematics be so taught that students will be able in later life to learn the mathematical skills which the future will surely demand of many of them. (6,3)

Thus, it would seem that the development of a statement of objectives and an evaluative instrument should be endeavors which complement each other, not activities that transpire independently of one another.

More basically, however, the purpose of education is to change students from a given state of experience to a desired state by means of a variety of appropriate learning experiences, some of which may be used as a basis for evaluation of achievement. To evaluate in this way, we measure at intervals and try to determine whether any change has occurred in the students. As time goes on, do they exhibit more or less of the desired behavior? Do they have deeper understanding of mathematical concepts? Do they have some concepts in June that they did not have in September? Do they appreciate mathematics more this year than they did last year? When we evaluate, we find answers to such questions, and we make judgments as to whether the amount and direction of change is satisfactory. We also render judgment as to whether the performance is typical for a given situation and whether it is indicative of maximum performance. Evaluation is the means we use to discover where we stand on the path between present experience and the objective. (15,23)

As Johnson and Rising put it:

In the construction of a new building, the architect does not design the building until he knows what effect he wishes to achieve; the builder does not select his materials until he knows what his blueprints specify; and the carpenter does not select a tool until he knows what operation he intends to perform. When the building is complete, it is considered acceptable only

if it satisfies the specifications established before it was built. In a similar way we must select materials that are appropriate for the program designed, and we must use the proper tools for the activity involved. Too often in the past teachers have discussed the relative merits of a new mathematics text, an instructional aid, of a teaching method without specifying just what goals they hoped to achieve with the materials being used.

If we do not know what our goals are, how can we evaluate a learner's progress or an experimental program or a new program? Only when we know where we want to go do we have a sound basis for selecting appropriate material, content, or instructional methods. Only when we know what progress has been made in attaining these goals can we evaluate the effectiveness of our instruction. (22,10)

Over the years many statements of goals or objectives have been formulated. These have varied from rather general statements of philosophical goals to detailed statements of student behavior relative to a particular topic or concept. The former is applicable to the development of an educational program for a state or even a local school district, but scarcely serve the needs of the classroom teacher whose day-to-day job it is to modify the behavior of those students who populate the elementary school classrooms. Statements of general goals frequently fail to mention the kinds of behavior one can expect to find when a child has mastered a particular concept. Thus, evaluation becomes difficult if not impossible. Students are often asked to do things on a test which are only tangentially related to what they were asked to do in class, or what they thought was the purpose of the learning experience.

Specific statements of objectives tend to keep the teacher from drifting too far afield from the task at hand.

. . . The first and most important step in planning a test is to define the objectives of instruction. (27, 35)

. . . It is the teacher's task to devise and to put into effect appropriate, challenging, fruitful experiences for children. Each experience should be designed to accomplish specific objectives or develop specific skills. This requires that the teacher have a clear conception of the goals and purposes of the school, the concepts to be developed, and the skills to be acquired. Whether long range or immediate, these should be such that they may be achieved and evaluated. Flowery, noble objectives that we cannot realize and that are not subject to testing or evaluation have little impact upon what transpires within the classroom.

Given a set of workable objectives, the teacher needs to create an environment in which specific concepts and skills may be developed. (28, 293)

Before any evaluation program is attempted, the teacher should carefully determine the objectives of the mathematics program deemed appropriate for the children in his class. When goals are clearly stated in behavioral terms--what the pupil should be able to do as a result of the experience--the extent to which the learner has progressed toward the desired objective can easily be determined. (35, 345)

When both what is to be taught and the behavior which should result from the instruction are included in the statement of the objective, the objective becomes the central point in the evaluation process. Consideration of the interrelatedness of objectives, instruction, and evaluation is necessary for effective teaching.

The entire process of evaluation requires the consideration of objectives, their relationship to one another, the instrument used to judge the accomplishment of the objectives, along with an analysis of the success or failure to attain the objectives.

The term 'behavioral objective' is used to identify a particular type of performance objective. It is interpreted as meaning a performance objective having an identifiable goal

which identifies the learner behavioral change that can be evaluated as a direct outcome of the goal. This type of objective is a two component statement written in exact terms which includes the first component, the goal, and the second component, the outcome or evaluation of the goal. Its key characteristic is that it not only states the goal but also identifies the instrumentation, performance, activity, or behavior the learner will become involved with as a means of evaluating the success he has achieved as an outcome of the intended goal of that particular objective. (35, 17)

Need for the Study

Early in 1967, the Wisconsin Department of Public Instruction published a booklet entitled Guidelines to Mathematics: K-6. (15) The objectives for the mathematics program for each of these seven grades were identified. The mathematical material for the elementary school is separated into fifteen general strands or topics. Under each of these is listed the specific behavioral objectives to be attained by the students in each of the grades kindergarten through the sixth grade. (The Guidelines to Mathematics: K-6 are listed as item fifteen in the bibliography. However, the specific behavioral objectives are included as Appendix A of this study.)

Because it was felt that no currently published test would evaluate the attainment of these objectives, a committee including representatives from the eight Wisconsin State Universities with laboratory schools, the University of Wisconsin at Milwaukee, and the Department of Public Instruction was formed in the fall of 1967. (See Appendix B for a list of committee members.) The primary function of this committee was to write an examination for each grade which would

evaluate the attainment of the objectives listed in the Guidelines.

These tests have now been completed and are available for use by the elementary school teachers in the State of Wisconsin. (The tests for grades one through six are included as Appendices C through H, respectively.)

Sound instructional procedures do not just happen. They are the result of careful, thoughtful planning. Good instruction is deliberately designed to support the objectives of instruction and to be consistent with the learning requirements of students. (7, 55)

Evaluation is necessary in any program, whatever the subject area. Evaluation should be concerned not only with the outcomes of instruction but also with the processes we are using. No matter how good we think our procedures may be they should be regularly evaluated to determine how well they are contributing toward students attaining our learning objectives. (7, 2)

Ultimately, continuing evaluation may well be the most important of all the characteristics of good instructional procedures. Evaluation helps us to make teaching and learning more effective and this, after all, is what we are seeking when we plan procedures of instruction. (7, 63)

It would seem that there is almost unanimous agreement that the first step in curriculum building is the development of a set of behavioral objectives; statements describing the modification of behavior of the individual student. Second, comes the teaching and the learning involved in the attainment of the specific objective. This includes a wide variety of teaching strategies which might be used. Third, seems to be the evaluation of the degree of attainment of the various specific behavioral objectives. At this point, the literature seems to express conflicting points of view. However, no one suggests

that an analysis of the test items as they relate to the behavioral objectives or to each other be undertaken. Are the objectives truly independent? Do the items test more than one objective? If they do, how does one determine what it is that the pupil does or does not know?

The question then would be, is it enough to establish a set of observable behavioral objectives, write and administer an examination purporting to evaluate the attainment of the objectives, and make some sort of judgement about the students success or failure? This seems to be only part of the picture. To complete it, one must analyze the items and the objectives to be sure that each is functioning in the manner it was supposed to. Thus, in building an evaluation instrument, the task is only partially completed when content validity, reliability, difficulty level, and a discrimination index have been determined. The need for further analysis is evident. This additional analysis will be completed in this study.

Definition of Terms

In general the meaning of each of the following terms and abbreviated forms of reference is indicated where it is first used in the report, but, for easy reference, the following list is presented at this point.

Achievement: accomplishment or proficiency of performance in a given skill or body of knowledge. (14, 7)

Achievement, academic: (1) knowledge attained or skills developed in the school subjects, usually designated by test scores or by marks assigned by teachers, or by both; (2) the achievement of pupils in the so-called "academic" subjects, such as reading, arithmetic, and history, as contrasted with skills developed in such areas as industrial arts and physical education. (14, 7)

Achievement, pupil: the status of a pupil with respect to attained skills or knowledge as compared with other pupils or with the school's adopted standards. (14, 7)

Behavioral Objective refers to a desired change in behavior, "The objective indicated a desire to change a student from a given state of experience to a desired state by means of a variety of appropriate learning experiences. (17, 23)

ELMR - 1 refers to the Elementary Mathematical Record or test for Grade One composed by the Laboratory School Mathematics Committee of Wisconsin.

ELMR - 2 refers to the Elementary Mathematical Record or test for Grade Two composed by the Laboratory School Mathematics Committee of Wisconsin.

ELMR - 3 refers to the Elementary Mathematical Record or test for Grade Three composed by the Laboratory School Mathematics Committee of Wisconsin.

ELMR - 4 refers to the Elementary Mathematical Record or test for Grade Four composed by the Laboratory School Mathematics Committee of

Wisconsin.

ELMR - 5 refers to the Elementary Mathematical Record or test for Grade Five composed by the Laboratory School Mathematics Committee for Wisconsin.

ELMR - 6 refers to the Elementary Mathematical Record or test for Grade Six composed by the Laboratory School Mathematics Committee of Wisconsin.

Evaluation: (1) the process of ascertaining or judging the value or amount of something by careful appraisal; (2) (psychs) the process of determining the relative significance of phenomena of the same sort in terms of some standard; (3) consideration of evidence in the light of value standards and in terms of the particular situation and the goals which the group or individual is striving to attain. (14, 209)

Evaluative criteria: (1) the standards against which a person or a group or a procedure may be checked; (2) the factors considered by an accrediting agency in analyzing the status of an educational institution to determine whether it should be accredited. (14, 209)

Measurement: (1) the comparison of a quantity (exhibited by a particular case) with an appropriate scale for the purpose of determining (within limits of accuracy imposed by the nature of the scale) the numerical value on the scale that corresponds to the quantity to be measured. (2) a term commonly applied to examining persons by giving some form of test; (3) the results obtained by measuring. (14, 337-338)

Standardized test refers to a test constructed and standardized by a commercial publishing company.

Strands or general strands refers to one or more of the fifteen general mathematical notions appearing as a general heading in Guidelines to Mathematics: K-6.

Purpose of the Study

Since parents and teachers alike want to know the level of achievement for each individual as well as each classroom group of children, behavioral objectives for mathematics for the elementary school have been written for the mathematics curriculum of the State of Wisconsin. The instrument used to identify the level of attainment of each of these behavioral objectives must be standardized. The purposes of this study were to determine whether or not the items on the test developed by the Wisconsin Laboratory School Mathematics Committee tested the concepts that the committee believed that they tested and to determine whether or not differences existed between groups of students because of differences in their schools.

The Problem

This study was designed to investigate the relationships which might exist between the general strands of objectives, between the specific behavioral objectives listed under one strand, and the specific behavioral objectives of one strand with those of a different strand.

The specific objectives of this study were:

1. To determine by means of a factor analysis the concepts of mathematics which the tests for each grade were testing.
2. To determine by means of a factor analysis which specific items, and hence, which specific behavioral objectives were related.
3. To determine whether or not there are differences between groups of students when classified according to (a) the size of the city or town in which the pupils attend school, (b) enrollment in the elementary school of the district, (c) per pupil expenditure for elementary school education, (d) per pupil valuation of the district, (e) the geographic location of the district in the State of Wisconsin, and (f) sex.

Hypotheses

In addition to using the factor analytic technique to determine the concepts of mathematics and the relationship of the individual test items to each other, the following six general null hypotheses will be tested:

1. There is no significant difference between group mean test scores for each grade as measured by the ELMR for that grade when classified according to the size of the city or town in which the pupils attend school.

2. There is no significant difference between group mean test scores for each grade as measured by the ELMR for that grade when classified according to the enrollment in the elementary schools of the district.
3. There is no significant difference between group mean test scores for each grade as measured by the ELMR for that grade when classified according to the per pupil expenditure for elementary school education.
4. There is no significant difference between group mean test scores for each grade as measured by the ELMR for that grade when classified according to the per pupil valuation for the district.
5. There is no significant difference between group mean test scores for each grade as measured by the ELMR for that grade when classified according to the geographic location of the district in the State of Wisconsin.
6. There is no significant difference between group mean test scores for each grade as measured by the ELMR for that grade when classified according to sex.

When the findings of the analysis of variance are presented in Chapter III, each of these general hypotheses will be restated so as to indicate the particular grade level under examination.

Delimitations

This study was limited to the objectives for elementary school mathematics as stated in the Guidelines to Mathematics: K-6, and to the test for each grade written by the Wisconsin Laboratory School Mathematics Committee specifically to test the objectives for that grade. The schools from which the students were chosen are all in the State of Wisconsin. All of the pupils were tested in late April or early May of 1969, with classroom groups being used.

Organization of the Study

The material presented in this study has been divided into six chapters. The first chapter was an introduction which included the background and setting for the writing of behavioral objectives for elementary school mathematics and the evaluation of the degree of achievement of these objectives, the need for the study, a listing of key terms and definitions, the purposes of the study, a statement of the problem with the hypotheses, delimitations of the study, and the scope of this study. The second chapter contained a summarization and the analysis of related literature and research. The third chapter discussed the methodology and design for the study. The fourth chapter presented the findings. The fifth chapter included the discussion. The final chapter of this study was a summary.

REVIEW OF THE LITERATURE

Introduction

During the past fifteen years, something closely akin to a "revolution" in mathematics has occurred. This revolution has resulted in changes in content, in curriculum, in methods of teaching, and in the evaluation of student achievement in the subject of mathematics. In order to facilitate the review of the literature, the material has been separated into three different categories: (1) comments from the literature, (2) review of published standardized tests, and (3) review of research. The material reviewed in the section entitled "Comments from the Literature" includes non-research type articles. In each of these, the author is expressing his opinion based upon his experience, reading, or convictions. Nothing in the way of research has been conducted.

In the section entitled "Review of Published Standardized Tests," the way in which the tests were constructed, analyzed, and standardized will be discussed.

Research in testing, test construction, development of statements of objectives, and the relationship of statements of objectives to evaluation will be examined in the section entitled "Review of Research."

The last section of this chapter is a summary of all of the material reviewed as it relates to the notion of the evaluation of

achievement in elementary school mathematics.

Comments from the Literature

The need for specifying behavioral objectives and for relating test items to the objectives has been emphasized by many authors.

As Dutton has stated, "The application of modern evaluation processes to the arithmetic program has been disappointingly slow and continues to be the most neglected aspect of arithmetic teaching."

(9, 58)

Nieder Korn, in 1971, said that "new programs and materials have been developed in great quantity during the past decade, but sound means of appraisal have not paralleled this sudden growth." (32, 2)

Hartung has this to say about evaluation:

More basically, however, the purpose of education is to change students from a given state of experience to a desired state by means of a variety of appropriate learning experiences, some of which may be used as a basis for evaluation of achievement. To evaluate in this way, we measure at intervals and try to determine whether any change has occurred in the students. As time goes on, do they exhibit more or less of the desired behavior? Do they have some concepts in June that they did not have in September? Do they appreciate mathematics more this year than they did last year? When we evaluate we find answers to such questions, and we make judgments as to whether the amount and direction of change is satisfactory. We also render judgment as to whether the performance is typical for a given situation and whether it is indicative of maximum performance. Evaluation is the means we use to discover where we stand between present experience and the objective. (17, 23)

According to Johnson and Rising, "If we do not know what our goals are, how can we evaluate a learner's progress or an experimental

program? Only when we know where we want to go do we have a sound basis for selecting appropriate material, content, or instructional methods. Only when we know what progress has been made in attaining these goals can we evaluate the effectiveness of instruction." (22, 10)

In 1946, one writer maintained that "any kind of learning can be diagnosed, whether or not that learning is important or desirable. If diagnosis is to occupy its rightful place in effective teaching, it is necessary that diagnosis be made in connection with worthy objectives of the educational program." (11, 380)

Hartung seems to be suggesting that the setting up of a set of objectives and writing a test covering them is not enough. "Thus, evaluation is needed in order to formulate an appropriate set of objectives and it is essential in order to determine how well the objectives, the desired changes in behavior, are being achieved." (18, 29)

In 1959, McLaughlin suggested that test publishers, to produce a standardized test, should first look at frequently used series of textbooks and courses of study. To determine major topics, the amount of content and time spent on a given topic should be determined. He would next solicit the advice of curriculum specialists in mathematics to determine "the educational objectives to be measured and the topics to be included in the test." (29, 6)

Taking note of the relationship of standardized tests to the objectives of arithmetic instruction, Spitzer argues:

Of the several shortcomings of tests which have been mentioned, we should be most concerned with the lack of agreement between content of the tests and the accepted goals or objectives of arithmetical instruction. One of the most important principles of evaluation is to base all evaluative procedures on the objectives of instruction. Now, if arithmetic tests are based on the objectives of instruction, it would seem that a reasonable picture of the goals of arithmetic might be deduced from the content of arithmetic tests. One has only to try that procedure with any of our present standardized tests to be convinced that arithmetic tests do not cover the whole field of instruction in arithmetic. (40, 23)

Bloom states that:

It is in the attempt to clarify and give operational definition to objectives that the construction of examinations is of critical importance. Educational objectives are frequently stated in such general form that the teacher who claims a particular objective has great difficulty in communicating what is meant by it to others, and perhaps as much difficulty in determining for himself exactly what was intended. At such a level of generality, the consequences for teaching must be almost nonexistent. At any rate, as the teacher describes the range of behaviors encompassed by the objective, it becomes possible to determine the kinds of observations, questions, problems, and tasks which it is appropriate to include in an evaluation instrument. (4, 389-390)

Review of Published Standardized Tests

Several of the writers' previously mentioned comments on the relationship of published standardized tests to the status of mathematics education in the elementary schools today. Let us turn our attention now to some of these standardized tests, more specifically to the manuals for the teachers which accompany each of them.

Every test publisher includes a list of instructions for administering the test. These vary from a very sketchy outline with a bare minimum of detail to a complete set including the words the

individual is to use in transmitting the instructions to the students taking the test.

Some manuals include a description of the way in which the test was constructed. In some cases this consists of little more than identifying content or topics to be tested, writing the test to incorporate these topics, and administering the test to a group of students to identify the various standards. In other cases, objectives were identified, items written, item analysis performed, and, in one instance, a factor analysis conducted to identify the significant factors.

The tests reported in this section were examined in the light of the following questions: (1) what was the source of the items included in the test? (2) were any behavioral objectives used in the construction of the items? (3) what item analysis was employed? (4) did the publisher include a discussion of the validity and reliability of the test? (5) was any past-testing analysis of the test conducted and reported?

In no way is the set of tests or manuals selected to be interpreted as complete. It is, however, representative of the tests which have been used to evaluate achievement in mathematics in the past.

In 1920, Monroe and Buckingham (30) published, *The Illinois Examination I and II*. Included are the Illinois General Intelligence Scale, Monroe's General Survey Scale in Arithmetic. The authors do not include any discussion of the manner of determining the items to be used in the test nor the topics covered in the arithmetic scale.

However, the authors discussed the validity, reliability, and objectivity of the test. They go to great lengths to discuss the interpretation of the scores on the various tests.

Orleans, in 1926, composed *The Public School Achievement Tests*. (33) Tests included covered the general areas of reading, arithmetic computation, arithmetic reasoning, language usage, spelling, grammar, history, geography, nature study, and health (physiology and hygiene) in three different batteries. In this instance, under the heading of validity, the author indicated the method by which he obtained questions for each test. The sources were questions constructed by teachers in making objective tests for use as final examinations for local school systems and the more recent and better textbooks in the elementary school subjects. Questions included work from grades above and below the one in which a student would be working at the time of the examination. Reliability and objectivity were also discussed. Again, there is no discussion of item analysis.

The authors of the *American School Achievement Tests* indicate that these tests "serve a four-fold purpose: (1) to measure pupil progress, (2) to assist in the classification of pupils, (3) to furnish data for a remedial reading program, and (4) to diagnose pupil's knowledge of specific concepts." (46, 2) However, the authors do not mention the manner in which the items of the arithmetic part of the test were constructed or selected for inclusion, except to say that they were "checked against commonly used textbooks and courses of study in the first grade." (46, 3) Validity and reliability were

discussed briefly. No mention was made of any specific objectives which were being evaluated.

In the examiner's manual for Test D of the Iowa Every Pupil Tests of Basic Skills, no mention is made of the item analysis nor of any relation of one item to another as a post-testing analysis. However, eight skills were identified and each of the items on the test was paired with a skill. These eight skills include "1, reading and writing numbers; 2, knowledge of common quantitative measures; 3, knowledge of numerical facts, terms and symbols; 4, identification of common geometric figures; 5, ability to make quantitative estimates; 6, ability to compare the size of numbers and fractions; 7, knowledge of the number system; 8, knowledge of common processes." (19, 14) As can be seen, most of these skills are very general and do not fit the definition of behavioral objectives. As an example, the skill "knowledge of numerical facts, terms and symbols" might include some questions involving any one of the four fundamental operations of addition, subtraction, multiplication, or division along with an interpretation of the symbols for "less than" or "greater than."

Joseph Jastak of the Delaware State Mental Hygiene Clinic, wrote the Wide Range Achievement Test in 1946. In the directions for administering this test, he makes no mention of the way in which the items were written, the manner in which they were chosen for inclusion in the test, an item analysis, any objectives which the items might be intended to measure, nor any follow-up evaluation of the test. Included are test age norms and corresponding test quotients.

The chief obstacle to a broad diagnostic program has always been the lack of achievement tests suitable for individual clinical work. Despite the availability of a large number of achievement scales the writer knows of no test which would fulfill the criteria of a good clinical test. The most important requirements of such a test are low cost, individual standardization, ease and economy of administration, suitability of contents, relevance of the functions studied, comparability of results over the entire range of the skills in question. The Wide Range Achievement Test was constructed with these factors in mind. (20, 3)

The directions for administering the Metropolitan Achievement Tests contain a fairly explicit discussion of "planning and experimentation undertaken to insure the validity, reliability, and efficiency of Metropolitan tests as measures of the important outcomes of the curriculum." (8, 21) (This same statement appears in the administrator's manual for each of the other batteries of this test.)

The Metropolitan Tests "attempt to measure those outcomes of instruction which, according to authoritative judgment and consensus of current practice, are the important goals of elementary instruction." (8, 21) The authors reviewed expert pronouncements, research, courses of study, and widely used textbooks to determine the goals. Next came the task of writing test items taking into consideration item type, appropriateness of directions, time limits, and related issues. Experimental forms of the tests were administered to pupils in nine school systems. The items were tried out in one or more grades above and below the grade in which they were intended. "On the basis of the actual performance of the items in the tryout, final forms of the various tests were developed that are of appropriate difficulty and range of difficulty, and that discriminate as effectively as possible

both among pupils in successive grades and among pupils of varying ability within each grade. The selection of items was carried on so as to produce equivalent final forms, each conforming to the original specifications established for the test. (8, 21)

Metropolitan norms purport to describe the achievement of pupils representative of the nation's public school population. Norming procedures were quite elaborate in order to arrive at this representative group. The sample chosen involved pupils from each state, from various sizes of schools, from rural and urban communities, various intelligence levels of the pupils, and segregated and non-segregated systems.

The purpose of the Stanford Diagnostic Arithmetic Test is to assist in the identification of learning difficulties encountered by pupils in the elementary grades. Since the authors are experienced in writing textbooks and workbooks, and in working with teachers and pupils, they were able to identify the major "trouble spots" used in the structure of the SDAT. In the preparation of the items the authors used them extensively in actual classroom situations until an adequate pool of items was available. (2)

An item analysis program was conducted in November, using data such as the mean raw score, split-half reliability, per cents of pupils in the total group, upper 27%, and lower 27% selecting each option for each item. This information was used to build the final forms of each subtest.

Standardization information includes norms, intercorrelations among subtests, reliability, and equivalence of forms. The relationship between Stanford Achievement Test: Arithmetic Tests and SDAT subtests was also determined.

No analysis of the items or factors other than an item analysis was reported.

The only reference to the way in which the Iowa Tests of Basic Skills were compiled is the statement that "The content of each test has been carefully selected to reflect the best of current curriculum practices. Only items of appropriate difficulty (as shown by preliminary tryouts) have been assigned to each grade level." (25, 3)

One difference between the information provided by the publishers of the Iowa Tests of Basic Skills and those of other tests is the pairing of each test item with an appropriate arithmetic concept as listed under a skills classification. These skills were not stated as behavioral objectives, rather as general classifications of content.

In the construction of the Modern Mathematics Concepts Test of the Stanford Achievement Test, the authors identified as a major goal "that the content of the test would be in harmony with present objectives of modern mathematics programs by the schools. . . . On the basis of this analysis the authors prepared detailed outlines of the content to be covered by the test at each grade level. These outlines specified the relative proportion of content to be devoted to the various skills, knowledges, and understandings within modern mathematics and served as blueprints for the tests that were

ultimately to emerge." (23, 7)

For each item in the experimental form, a percent of the pupils at each grade level answering the item correctly and the percent selecting each incorrect choice for each item was obtained. For each item, the discrimination index was also computed.

Each item in the test is identified as testing one of several general mathematical topics. The authors indicate that some items may test more than one category, they are, nevertheless, placed in only one category.

The reliability of the test was calculated two different ways, the odd-even split-half method and by an estimate of the Kuder-Richardson Formula 20 reliability coefficient.

Madden and Peak, (26) decided to limit the content of their test to the measurement of the competence in the fundamental operations on whole numbers, common fractions, decimals, and per cent. Items were submitted to groups of mathematics teachers for comment. In making the final selection of items, the authors considered difficulty, i.e., per cent of students getting the correct answer, and the item-test validity, i.e., the extent to which success on the item agreed with success on the entire test.

Standardization, validity, reliability, and standard errors of measurement were calculated for each part as well as for the entire test.

Also, for this particular test a variety of intercorrelations

were calculated. These were completed to show the degree of relationship between the various parts of the test. As might be expected, these indicated considerable correlation.

The manual of directions for the Hundred Problem Arithmetic Test states that "Trained educators and skilled teachers have long felt the need for a reliable standardized test of computational ability which can be given in one class period. More recent trends in the development of test theory and practice toward the separate measurement of independent mental abilities, rather than composite measurement where many abilities enter to an unknown degree make it even more important at this time to have a separate measure of numerical facility, such as is provided in the Hundred Problem Arithmetic Test." (37, 1) No mention is included about the basic construction of this test nor of any statistics regarding any of the items chosen for inclusion in the test.

After reviewing professional books, articles, conference reports, yearbooks of authoritative associations, research reports, courses of study, teachers' guides, and modern text materials, the authors of the SRA Achievement Series (44) developed a list of nineteen broad objectives or skills under five general headings. It must be noticed that these "objectives" are not stated in today's behavioral fashion. Nonetheless, some statement of a concept to be mastered by the learner is indicated. The authors indicate the skill or skills being tested by each item. However, the authors, in this particular publication, fail to discuss any statistical analysis of either the

items or the test as a whole. One must go to a different manual to find this information. There one sees a very complete description of the development of the SRA Achievement Series.

The authors surveyed important statements of the overall purposes of education, publications of authoritative groups and specialists, and courses of study and curriculum guides. These lead to the establishment of three criteria for the final selection of the major areas to be tested:

1. The objectives to be measured must be amenable to description in terms of observable behavior and must also be amenable to objective measurement.
2. The behaviors to be measured must include only those for which valid and reliable data can be secured in a paper-and-pencil test situation.
3. The test must be as rich in sampling of behaviors in life-like situations as time limits permit. (33, 3)

In the analysis of the items of the SRA Achievement Test, we find one of the most complete attempts to be identified in the literature. Not only are the usual discrimination index, difficulty index, reliability coefficients, and validity computed, but a factor analysis is also completed. This was done in order to identify any existing pattern of relationships which might exist among the scores. A general achievement factor and four group achievement factors were extracted.

Review of Research

A number of studies have been completed which deal with behavioral

objectives, evaluation, and the construction of an evaluative instrument was to identify the specific behavioral objectives to be tested.

In others, the first step was to identify concepts without any statements of specific behavior. Invariably the next step was to write a series of items which would test the behavior or the concept, whichever was the case in the first step. In some instances, this was as far as the study went. The evaluative instrument was considered complete. In others, an item analysis, usually consisting of the calculation of a discrimination index, a difficulty index, and a coefficient of reliability was performed. More often than not, content validity was determined by a panel of experts in elementary school mathematics education.

In a few instances, a factor analysis was also completed. This was done to identify common factors or to determine the uniqueness of each of the items on the test.

The studies reviewed in this section are those found in the dissertation abstracts or among the ERIC documents. These two sources proved to be the most useful after the standardized tests had been examined.

ERIC Documents

Alkin defines evaluation as "the process of ascertaining the decisions to be made, selecting related information, and collecting and analyzing that information in order to report summary data useful to decision makers in selecting among alternatives. (1, 3-4) He

further goes on to state:

Behavioral objective specification is not necessarily a panacea for evaluation of all types. While all enterprises should have a goal, these goals are not necessarily always specifiable in student behavioral terms. Moreover, when complex systems are involved the specification of objectives in behavioral terms, has been somewhat overplayed in its relative importance. I would submit also that the broadening definition of evaluation has considerably modified views about the need for specification of behavioral objectives. (1, 2)

Skager says that "the need to refine and standardize measures of cognitive skills is related to a more pragmatic and immediate concern, the evaluation of educational programs." (37) He goes on to list three reasons by which the measurement of cognitive skills can make the conclusions in the evaluation for more meaningful.

Differences between norm-referenced measures and criterion-referenced measures as to the information they provide are identified by Klein. (24) Norm-referenced data relates one individual's score to those of others on the same instrument while criterion-referenced data shows an individual in relation to some performance standard. The latter helps teachers decide whether their program is attaining the objectives.

Dissertations

The purpose of Rappaport's study was to determine "the degree of children's understanding of basic concepts and operations from the number system. The abstract did not include any information about the analysis of the items or of the test. (34)

Whitman (45) developed a test of conceptual knowledge of arithmetic.

Examination of texts provided the topics to be tested. In particular, those which were most commonly identified were selected for inclusion in the test. Validity and reliability were both determined. No behavioral objectives were written from which test items might be constructed. She did not conduct a factor analysis to be sure that the items were testing the concepts initially intended.

The construction and standardization of an evaluative instrument designed to measure the degree to which children in grades three, four, five, and six comprehend certain selected principles of the mathematics program was the purpose of the research of Edwards. (10) No statement of behavioral objectives was included but the following five general concepts were tested: 1) numeration system, 2) commutative principle of addition and multiplication, 3) associative principle of addition and multiplication, 4) distributive principle of multiplication over addition and subtraction, and 5) identity element of zero in addition and subtraction and of one in multiplication and division. Reliability was determined by Froelich's formula while validity was determined by a panel of nineteen experts in the field of arithmetic instruction. Indices of discrimination and difficulty were calculated. No mention was made of any factor analysis.

Garner (13) also constructed a test to measure the understanding of basic concepts and principles in elementary school arithmetic. These were to be in harmony with the objectives of the new curricula. The objectives were not identified in the dissertation but five general

areas were: 1) number and numeration, 2) number operations and processes in addition and subtraction of whole numbers, 3) number operations and processes in multiplication and division of whole numbers, 4) meanings and processes with fractions, and 5) meanings and processes with decimals. To select the items for the final form of the test, an item analysis to determine difficulty and discriminatory power was completed. Reliability was determined by the Kuder-Richardson Formula Twenty. Content validity was established by the construction of items to measure concepts considered basic by authorities in the field of arithmetic. Concurrent validity was established by correlating scores on the researcher's test with scores on the Metropolitan Achievement Test in Arithmetic.

Hartlein (16) constructed a test of mathematical understandings using two kinds of items, some using number words and some using coding. She evaluated the items for difficulty and discriminatory power.

In his analysis of standardized test items, Bernabei (3) chose two standardized arithmetic achievement tests; the California Achievement Test, Form W, 1957 and the Iowa Test of Basic Skills, Form 1. He identified the objectives of the School Mathematics Study Group program. Using a two-way grid, the taxonomy on the vertical scale and the SMSG objectives on the horizontal scale, a logical analysis of the test items was completed. The author, using his own judgement, placed each item in a cell on the two-way grid. No factor analysis was conducted to assist him in this endeavor.

A test of fifty multiple-choice items to measure understanding of the real number system was constructed by Jensen. (21) An item analysis including a difficulty index and a point biserial correlation to determine the ability of each item to discriminate between poor and good students as determined by the total score on the test was completed. Reliability by the split-half technique and validity by a panel of authorities were determined. Four basic categories were identified for testing: 1) sets, 2) factors and primes, 3) operations, and 4) number systems. No factor analysis was conducted.

Gallian (12) conducted a content validity study to determine which of four achievement tests might best be used with six of the most frequently used arithmetic series in the state of Missouri. He concluded that tests were very similar in skills and concepts covered but that they covered a limited number of skills and concepts presented in the arithmetic texts which he analyzed.

Another study whose purpose was to develop an instrument to measure achievement in modern mathematics in the primary grades was conducted by Thompson. (42) To obtain the objectives to be tested, Thompson surveyed courses of study, modern mathematics textbooks, and reports of recognized study groups. He computed validity, reliability, level of difficulty, and a discrimination index. Expert opinion instead of factor analysis was used to relate items and objectives.

Although the primary purpose of Morford's study (31) was not the construction of a test, he did construct such a test based on the behavioral objectives identified for an education course and classified

according to Bloom's Taxonomy. He predicted that cognitive process factors would emerge from factor analysis of responses on the Taxonomy-type instrument. However, factor analytic procedures did reveal the four cognitive process factors or five logical factors of taxonomical behavioral objectives. Ten factors were identified and the objectives of the education course were assigned to these.

Boyden (5) constructed a computer scored diagnostic test in verbal problem solving. Twelve error categories were identified. Each incorrect response was keyed to the category with which it was associated for analysis using the computer. Item difficulty and discrimination along with the reliability of the test were calculated.

Summary

A number of tests covering the field of elementary school arithmetic have been written, both as doctoral research projects and as commercially published ventures. The analysis of the tests and the items contained in them varies from none at all to rather complicated and complete factor analyses.

Another aspect of the testing situation has to do with the content of the instrument. In some cases the authors identified general topics and constructed items which would be compatible with those general topics. In other cases the authors were concerned with the identification of student behavior and wrote items for a test after first indicating specific behavioral objectives.

As one looks at copyright dates, one sees that as learning theory has become more sophisticated, so has evaluation theory. No longer is it possible to sit down and whip out a test and say that the test evaluates the achievement of the elementary school student. It has become necessary to indicate specifically what each item as well as the whole test is testing. Specific behavior by the students must be pinned down. Consumers of tests want to know fairly definitely what it is that they are buying.

METHODS AND PROCEDURES

Introduction

This study was designed to investigate the relationship that exists between objectives for elementary school arithmetic as stated in the Guidelines to Mathematics: K-6 published by the Department of Public Instruction of the State of Wisconsin. The investigation also considered the relationship between test scores of pupils because of the size of city or town, the expenditure per pupil for elementary school education in a district, the enrollment in elementary schools, the valuation per pupil in the district, the geographic location of the district in the State of Wisconsin, and the sex of the pupils.

Tests were written with the items designed to test a particular behavioral objective. These were administered in late April and early May of 1969 to the classrooms selected for the sample. The answers to the questions and the scores on the tests will be used as the basis for the analysis in this investigation.

The Objectives

In the Guidelines to Mathematics: K-6, the mathematical concepts of the elementary school curriculum have been organized under fifteen main topics: 1. Sets and Numbers; 2. Numeration Systems; 3. Order; 4. Number Systems, Operations and Their Properties; 5. Ratio and Proportion; 6. Computation; 7. Size and Shape; 8. Sets of Points; 9. Symmetry; 10. Congruence; 11. Similarity; 12. Coordinate Systems

and Graphs; 13. Constructions; 14. Measurement; and 15. Mathematical Sentences.

Under each of these main topics are listed the specific behavioral objectives deemed appropriate for a particular grade. The number of these vary from topic to topic and from grade to grade. For the seven grades, a total of two hundred seventy-six different objectives are listed. For the grades one through six, a total of two hundred sixty-three are included, thirty-one for grade one, forty-two for grade two, forty-five for grade three, forty-seven for grade four, fifty-three for grade five, and forty-five for grade six. Not all of the fifteen main topics are appropriate for each of the grades, kindergarten through six. For instance, pupils in the kindergarten or first grade are not expected to be able to fully understand the notions of ratio and proportion or congruence. Therefore, no specific behavioral objectives are listed under these headings for those grades. (The Guidelines to Mathematics: K-6 are listed as item fifteen in the bibliography. However, the specific behavioral objectives are included as Appendix A of this study.)

The placement of the topics and the order in which the general topics and specific behavioral objectives are listed is not to be considered as the only correct arrangement of these concepts, nor the correct order in which they should be presented. "The ideas listed for each grade level should be regarded as a suggested guide for introducing various topics; the outline is not intended to be all-inclusive." (15, 5)

The Tests

As stated in Chapter 1, tests which are considered to evaluate the degree of attainment of a specific objective have been written for grades one through six. Each objective for each grade was examined by the test committee to determine whether or not it stated an observable, measurable, behavior. A few, such as the fourth one under the strand, Numeration Systems, for the sixth grade were deemed not observable, behavioral objectives. Each committee member and the appropriate teacher from the laboratory school of the institution at which the committee member was teaching, wrote a number of possible test items for each objective. Test items were to be of the multiple-choice type with four choices. They were to contain language appropriate for the intended grade level.

The items were presented to the entire test committee for consideration. Each item was examined for content validity. Each distractor was examined for plausibility. Some items were rejected because they did not meet one or both of these conditions.

Items accepted were put in the form of a test, mimeographed for distribution and administered to the appropriate grade in some of the laboratory schools. Each item was then reconsidered by the test committee. A discrimination index using the formula $D_I = \frac{N_H - N_L}{N}$, where N_H is the number of pupils in the highest one-fourth of the test group answering the item correctly, N_L the number of pupils in the lowest one-fourth of the test group answering the item correctly, N is one-

fourth of the number of pupils to whom the test was administered and D_I is the discrimination index which resulted from the calculations. A discrimination index of greater than .40 was deemed satisfactory for the purposes of the tests. Items for each objective were reconsidered. If several items testing a particular objective were found to have a discrimination index in the acceptable range, those which did not were placed in a "hold" file for possible later reconsideration. The acceptable items were then examined to see if each of the incorrect responses was functioning as a distractor. If a response was not functioning, the item was reconsidered and placed in reserve for possible later modification to increase its acceptability.

The items remaining in the acceptable list were examined from the standpoint of difficulty. Items answered correctly by everyone were considered unacceptable. So were items answered incorrectly by every pupil. These were scrutinized in an attempt to determine the reason for every pupil choosing an incorrect response. Such information might be helpful in making the acceptable items stronger. The difficulty level is simply the per cent of the pupils taking the test who answered the item correctly. Items answered correctly by more than 80% were considered too easy while those answered correctly by less than 20% were considered too difficult. Any item within the range of 20% to 80% remained on the acceptable list.

If no items testing a specific objective were considered acceptable, the previously discarded items were reconsidered. If a

modification was possible which, in the opinion of the members of the committee, would make an item acceptable, it was made. These items were incorporated into a test and administered to a different group of pupils in the laboratory schools. The process of computing a discrimination index and difficulty level, and looking for distractors not used was completed again. The entire process was redone until three or four items for each objective were considered to be acceptable.

These items were put in two different forms of a test for each grade and administered to a group of pupils chosen from the cities in which the campus is located or communities close to the campus. No attempt was made to choose these pupils on a statistically sound basis. Ease of delivering the tests to the schools was the most important factor.

Upon examination of the results of this administration of the test, the items considered to be the best for testing a particular objective were incorporated into a test for each of the grades. Thus, six different tests were completed. These were administered in late April or early May of 1969 and the results used for the current study.

The Students

Since a different test was constructed for each of the grades, one through six, different groups of pupils were needed. Classroom units rather than specific pupils were to be used for the administration of the tests.

Most of the population of the State of Wisconsin is located in the southeast portion of the state. As one proceeds in a northwesterly direction, the cities and towns become smaller and further apart. School districts become larger from the standpoint of square miles within the district, but smaller from the standpoint of enrollment.

In the selection of students to participate in the study, a number of factors were considered so as to get a representative sample of the elementary school population. These figures were supplied by the Department of Public Instruction of the State of Wisconsin.

The cities and towns in the state were identified as being small, medium, or large, with Milwaukee in a category by itself. A small community was one with a population of less than five thousand residents. A medium community was one with more than five thousand but less than twenty thousand. A large community was identified as having more than twenty thousand but less than two hundred thousand residents. Milwaukee has a population of over seven hundred thousand residents so it was put into a category by itself.

In the second sort the schools were categorized according to the expenditure per pupil for elementary school instruction. Three categories were used; high, over \$800 per pupil; and medium, between \$600 and \$800; and low, less than \$600.

The third sort was based on the enrollment in the elementary schools. Again, three categories were used: large, an elementary

enrollment of over five thousand pupils; medium, an elementary school enrollment of between one thousand five hundred and five thousand pupils; and small, schools with an elementary school enrollment of less than one thousand five hundred.

The valuation per pupil of the district was the basis of the fifth sort. This was also separated into three categories: high, a valuation of more than \$34,000 per pupil; medium, a valuation of between \$20,000 and \$34,000 per pupil; and low, a valuation of less than \$20,000 per pupil.

The last sort was perhaps the most difficult of all, mainly because of the uneven distribution of the population in the State of Wisconsin. The final decision was made to have a rather large northern section, with an irregular boundary, a smaller central section, and a small southern section. No attempt was made to balance the sections from the standpoint of population.

Collection of the Data

As previously stated, the tests were administered to the classes in late April or early May of 1969, depending upon how this fit into the school district's educational program. Each pupil was encouraged to attempt each item. No time limit was established for any of the tests.

In every case, the tests were delivered to the schools and the answer sheets collected by a member of the test committee. Each classroom teacher administered the tests to her own class.

Treatment of the Data

Because the pupils of grades one and two marked the answers on the test booklets, their answers were transferred to machine scoreable answer sheets. These, along with the answer sheets for grades three through six, were then machine scored and the answers transferred to data cards.

Very few of the students ignored questions. However, a few of them did not answer some of the questions. Omissions were treated as incorrect responses.

One of the purposes of this study was to identify the factors being tested by each of the tests. The procedure used was to compute Pearson product-moment correlation coefficients among all of the items of each test. The largest correlation of an item with one of the remaining items was placed in the diagonal of the correlation matrix and factors were extracted by principal components. These factors were rotated first to a Varimax solution and inspected for meaningfulness. This is a judgemental process whereby one seeks to find a common attribute among the items loading similarly on the factors. At this point, the item difficulties were compared with the factor loadings in order to identify difficulty factors. Finally, a Procrustes rotation was performed using Wherry's procedure. (45)

To test the hypotheses relating to size of city or town, per pupil expenditure for elementary school education, enrollment in elementary schools, valuation per pupil in the district, geographic

location in the State of Wisconsin, and sex, the data was analyzed using an analysis of variance technique. In determining the degrees of freedom, the test scores of each individual pupil rather than classroom units or schools were used. For each significant value of F, the t-test procedure was used to test the significance of the difference between pairs of group mean test scores.

FINDINGS

This chapter is comprised of several sections which relate the findings and analysis that resulted from the statistical treatment of the data. The major sections of the chapter are: (1) the factor analysis of the test items for each grade, and (2) an analysis of variance technique for each grade to answer the six questions asked about the students.

The Factor Analysis

Pupils in each of the grades from one through six in schools in Wisconsin were administered an appropriate test based upon the objectives identified in Guidelines to Mathematics: K-6. To analyze the test items independent factor analysis were conducted for each grade.

Grade one factor analysis

The factor analysis for grade one indicated the elementary concepts of mathematics customarily taught to first grade pupils. There were eight factors as shown in Table 1. These eight factors accounted for 22.23 per cent of the total variance.

The indicators of the variance assumed by each of the test items, h^2 , in Table 1 demonstrates that the items were relatively inconsistent, ranging from .06 for item thirteen to .49 for items thirty and fifty-nine. This may be an indication that some of these concepts were not taught to some of the pupils, either by teacher

Table 1. Rotated factor loadings for grade one

Items	I	II	III	IV	Factors V	VI	VII	VIII	h^2
1	-.09	.14	.10	.29	-.01	.04	-.13	.09	.15
2	-.04	.15	.06	.16	-.10	.10	-.32	.16	.20
3	-.07	.59	.08	.09	-.01	.10	-.17	.12	.42
4	-.21	.10	.12	.15	.05	-.03	-.46	.18	.34
5	-.12	.00	.04	.06	-.06	.12	.01	.34	.15
6	-.02	.06	.05	.26	.04	.04	-.11	.45	.29
7	-.14	.05	.09	.00	-.06	-.01	-.16	.34	.18
8	-.16	-.01	-.01	.22	-.09	.01	-.22	.14	.15
9	-.23	.23	.03	.30	-.06	.08	-.15	.27	.30
10	-.10	.07	.04	.03	.00	-.07	-.10	.36	.16
11	-.05	-.00	.09	.02	-.11	.06	-.13	.29	.13
12	-.13	.07	.05	.04	-.08	.07	-.37	.27	.25
13	-.10	.03	.12	.13	.02	.02	-.09	.09	.06
14	-.05	.14	.09	.05	-.10	.57	-.19	.09	.41
15	-.10	.03	.11	.44	-.14	-.05	-.03	-.01	.24
16	.05	.06	.05	.29	-.05	.13	-.06	.11	.13

Table 1 (Continued)

Items	I	II	III	IV	Factors V	VI	VII	VIII	h^2
17	-.08	-.10	-.03	.32	-.01	.06	-.19	.07	.16
18	.02	-.01	.11	.17	-.01	.10	.02	.16	.08
19	-.05	-.01	.08	.07	-.48	.09	-.06	.13	.27
20	-.08	.02	.07	.07	-.51	.08	-.02	.12	.30
21	-.02	.09	.03	.19	-.17	.07	-.14	.14	.12
22	-.17	.21	.08	.15	-.13	-.03	-.29	.06	.21
23	-.13	.13	.02	.18	-.20	-.01	-.18	.08	.15
24	-.10	.18	.10	.20	-.15	-.10	-.05	-.03	.13
25	-.32	.09	.10	.11	-.16	.03	-.14	.19	.22
26	-.23	.03	.29	.12	-.14	.09	-.35	.18	.33
27	-.14	.07	.08	.09	-.16	.18	-.27	.23	.22
28	-.15	.07	.48	.01	-.04	.10	-.09	.17	.31
29	-.13	.06	.65	.12	-.06	.05	-.12	.10	.48
30	-.09	.09	.65	.14	-.06	.04	-.15	.10	.49
31	-.20	.04	.55	.19	-.10	-.01	-.12	.11	.41
32	-.22	.08	.07	.09	.04	-.01	-.07	.10	.09

Table 1 (Continued)

Items	Factors								h ²
	I	II	III	IV	V	VI	VII	VIII	
33	.06	.01	-.01	.13	-.05	.14	-.23	.10	.11
34	-.07	.61	.13	.06	-.12	.07	-.20	.12	.47
35	-.22	.11	.14	.04	-.06	.05	-.49	.17	.35
36	-.08	.00	.09	-.00	-.11	-.01	-.10	.31	.13
37	-.11	.07	.13	.20	-.03	.05	-.12	.36	.22
38	-.11	.11	.07	.06	-.14	.05	-.04	.22	.10
39	-.21	-.05	.00	.11	-.05	.09	-.20	.21	.15
40	-.20	.12	.08	.27	-.04	.09	-.07	.29	.23
41	-.21	-.06	.06	.24	.00	.04	.01	.30	.20
42	.11	-.05	.02	-.05	.02	-.04	.07	-.23	.08
43	-.20	.19	.09	.11	-.07	.10	-.11	.19	.16
44	-.14	.06	.06	.26	-.09	.02	-.25	.08	.17
45	-.11	.13	.01	.05	-.16	.56	-.22	.05	.42
46	.02	-.01	.04	.08	.00	.03	-.17	.13	.05
47	.02	-.06	.01	.16	-.03	.11	-.09	.18	.08
48	.01	.02	.12	.06	-.07	.09	-.20	-.01	.07

Table 1 (Continued)

Items	I	II	III	IV	Factors V	VI	VII	VIII	h^2
49	-.03	.16	.10	.05	-.13	.03	-.31	.15	.17
50	-.01	.04	.07	.05	-.40	-.02	-.16	.12	.21
51	-.04	.07	.00	.03	-.31	.00	-.14	-.05	.13
52	-.07	.04	-.00	.12	-.20	.03	-.30	-.05	.15
53	-.01	.11	.03	-.01	-.11	.05	-.27	.07	.10
54	-.24	.07	.05	.26	-.12	.05	-.15	-.05	.18
55	-.02	.03	.10	.25	-.07	-.08	.04	.12	.10
56	-.29	.01	.13	-.01	-.15	.04	-.33	.19	.27
57	-.23	.06	.15	.01	-.05	.05	-.42	.08	.27
58	.04	.00	-.06	.03	-.01	-.08	.08	-.01	.02
59	-.64	.02	.13	.06	-.13	.06	-.07	.20	.49
60	-.38	.07	.29	.24	-.09	.08	-.09	.24	.38
61	-.49	.04	.19	-.01	-.06	.06	-.23	.24	.40
62	-.63	.01	.17	.07	-.07	.07	-.11	.18	.48

decision or by textbook organization, or that the concept was difficult for the first grade pupils.

Factor I The items that loaded on this factor were all negative loadings. The combination of items accounted for 3.69 per cent of the variance. The items loaded on Factor I were:

Item	Factor Loading
59	-.64
62	-.63
61	-.49
60	-.38

These items all involve basic subtraction facts, facts which all textbooks for elementary school mathematics expect the students to understand and commit to memory. Factor I was called the basic subtraction factor.

Factor II The items that loaded on this factor were all positive loadings. The combination of items accounted for 1.94 per cent of the variance. The items loaded on Factor II were:

Item	Factor Loading
34	+.61
3	+.59

These items involve the insertion of a set which is missing between two given sets. Factor II was called the insert the missing set factor.

Factor III The items that loaded on this factor were all positive loadings. The combination of items accounted for 3.10 per cent

of the variance. The items loaded on Factor III were:

Item	Factor Loading
29	+ .65
30	+ .65
31	+ .55
28	+ .48

These items all involve basic addition facts which are expected to be mastered by all grade one students. Factor II was called the basic addition factor.

Factor IV The items that loaded on this factor were all positive. The combination accounted for 2.53 per cent of the variance. The items loaded on Factor IV were:

Item	Factor Loading
15	+ .44
17	+ .32
9	+ .30
16	+ .29

With the exception of item nine, these items all involve the naming of geometric figures. Item nine involves writing a numeral after hearing the word for that numeral. Factor IV was called a geometric factor.

Factor V The items comprising this factor accounted for 2.06 per cent of the variance. The items loaded on Factor V were:

Item	Factor Loading
20	-.51
19	-.48
50	-.40
51	-.31

Each of these items involves writing the number sentence which relates to a number line with arrows drawn on it to indicate direction and amount of each grouping. This factor was called the writing the number sentence factor.

Factor VI The items comprising this factor accounted for 1.57 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
14	+.57
45	+.56

Each of these items involves the relationships between addition and subtraction. The pupils were given a number sentence involving one of these operations and were required to write a sentence using the same numerals but the opposite operation. Factor VI was called the relationship between addition and subtraction factor.

Factor VII The items comprising this factor accounted for 3.87 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
35	-.49
4	-.46
57	-.42

Since each of these items involves renaming numbers, this factor was called the renaming factor.

Factor VIII The items comprising this factor accounted for 3.48 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
6	+.45
10	+.36
37	+.36
5	+.34

Each of these items involves identification of some place value concept. Therefore, Factor VIII was called the place value factor.

Grade two factor analysis

The factor analysis for grade two is shown in Table 2. It indicates the basic mathematical concepts usually presented to second grade pupils. There were ten factors which accounted for 23.77 per cent of the variance.

The indicators of the variance assumed by each of the test items, h^2 , in Table 2, demonstrates that the items were relatively inconsistent, ranging from .03 for item seventy-two to .65 for items three and thirty-nine. This may be an indication that some of these concepts

Table 2. Rotated factor loadings for grade two

Items	I	II	III	IV	V	Factors VI	VII	VIII	IX	X	h^2
1	-.24	.02	.43	-.12	-.00	-.12	-.02	-.08	-.01	.14	.30
2	-.08	.16	.11	-.27	.03	.04	-.10	-.07	.20	.01	.17
3	-.15	.23	.69	-.06	.04	.04	-.05	-.21	.16	.14	.65
4	-.07	.12	.16	-.19	.11	-.01	-.10	-.33	.05	.11	.23
5	-.15	.25	.12	-.05	.01	.14	-.13	-.40	.02	.05	.30
6	-.19	.10	.15	-.26	.02	.09	-.04	-.06	-.09	.16	.18
7	-.41	.19	.18	-.13	.12	-.11	.02	-.23	-.08	.05	.34
8	.05	-.03	.07	-.40	.02	.07	-.05	-.16	.07	.13	.22
9	-.04	-.03	.08	-.22	.06	-.01	-.18	-.17	-.02	.27	.19
10	-.07	.39	.19	-.04	.06	-.18	-.27	-.32	-.04	-.02	.40
11	-.25	.04	.11	-.13	.02	-.13	-.25	-.19	-.05	.14	.23
12	-.19	.09	.08	.01	.07	-.08	-.11	-.24	.08	.18	.17
13	-.02	.20	.01	-.01	-.01	.04	.01	-.09	-.00	.36	.18
14	-.09	.24	.09	.02	.08	.40	-.04	-.18	.06	.07	.28
15	-.21	.25	.08	-.10	-.02	-.04	.01	-.27	.38	.18	.38
16	-.30	.13	.07	-.27	.10	-.02	-.10	-.16	.08	.12	.25

Table 2 (Continued)

Items	Factors										h^2
	I	II	III	IV	V	VI	VII	VIII	IX	X	
17	-.04	.19	.06	-.01	.61	.10	-.09	-.11	.10	.15	.48
18	-.28	.25	.02	-.03	.04	.11	-.07	-.19	.34	.22	.36
19	-.13	.29	.02	-.09	.11	-.08	.04	-.08	.04	.10	.15
20	-.02	.21	.06	-.05	.07	-.04	-.04	-.06	.08	.21	.11
21	.01	.09	.02	-.09	-.13	-.03	-.21	-.03	.07	.20	.13
22	.03	.27	.12	-.14	-.01	.10	.04	-.11	.09	.07	.14
23	-.03	.14	.07	-.09	.01	-.02	.06	-.08	.03	.17	.07
24	-.12	.31	.16	-.14	-.02	-.01	-.05	-.12	.01	.11	.19
25	-.15	.14	.12	-.24	.14	.05	-.01	-.11	.10	-.01	.16
26	-.22	.05	.08	-.14	.08	-.06	-.09	-.37	.01	.01	.23
27	-.20	.06	.05	-.21	.08	.09	-.24	-.32	-.08	-.01	.27
28	-.15	.05	.18	-.18	.04	.10	-.10	-.34	.05	.07	.24
29	.02	.09	.08	-.06	.06	.03	-.00	-.44	.00	.15	.24
30	-.05	.06	.14	-.06	.06	.02	.01	-.34	.22	.08	.20
31	-.24	.11	.08	-.17	-.00	.10	.06	-.45	.04	-.03	.32
32	-.22	.19	.10	-.00	.06	.07	-.01	-.23	.13	.15	.20

Table 2 (Continued)

Items	Factors										h ²
	I	II	III	IV	V	VI	VII	VIII	IX	X	
33	-.03	-.00	-.06	-.10	.10	.15	-.04	-.10	-.02	.18	.09
34	-.05	.24	.04	.01	.14	-.04	.08	-.22	-.03	.25	.20
35	.04	.06	.04	-.07	.07	.07	-.25	.01	.03	.02	.09
36	-.25	.14	.06	-.01	-.05	.11	-.14	-.01	.06	.05	.12
37	-.06	.09	.05	-.06	-.02	-.16	.06	-.03	-.00	.17	.08
38	-.08	.27	.66	-.12	-.02	.17	-.05	-.18	-.00	-.00	.59
39	-.12	.20	.70	-.07	.01	.07	-.07	-.28	.12	.06	.65
40	-.05	.07	.00	-.50	-.00	.00	-.00	-.07	.01	-.01	.26
41	-.04	.41	.08	-.03	-.01	.14	-.09	-.28	-.03	.02	.30
42	-.01	.03	.07	-.35	-.03	-.11	-.11	-.18	-.04	.22	.23
43	-.02	.10	.05	-.03	.05	-.11	-.04	-.24	-.12	.21	.15
44	-.05	.18	.05	.01	.60	.04	-.06	-.22	.10	.17	.49
45	-.10	.24	.10	-.02	.03	.15	-.06	-.33	.21	.18	.29
46	-.12	.35	.05	-.11	.05	.13	-.09	-.06	.16	.18	.24
47	.06	.03	.09	-.00	.01	-.04	-.18	-.09	.11	.15	.09
48	-.24	.02	-.04	-.06	-.09	.01	-.48	-.27	.05	.06	.38

Table 2 (Continued)

Items	Factors										h ²
	I	II	III	IV	V	VI	VII	VIII	IX	X	
49	-.31	.12	.07	-.01	-.03	.05	-.20	-.38	.04	.02	.30
50	-.34	.01	.07	-.19	.02	.03	-.34	-.35	.09	.05	.40
51	-.15	.17	.07	-.11	.02	.08	.01	-.41	-.07	.02	.24
52	-.36	.10	.06	-.02	-.02	.03	-.04	-.13	.06	-.00	.17
53	.00	.19	-.01	-.17	.04	-.01	-.10	-.45	.08	.01	.29
54	-.03	.37	.10	-.06	.08	.18	-.09	-.05	.05	.25	.27
55	-.16	.32	-.00	-.02	.20	.08	.01	.07	.02	.16	.20
56	-.10	.37	.07	-.02	.03	.09	.05	-.05	.00	.06	.17
57	-.36	.16	.14	-.12	.07	-.02	.06	-.14	-.08	.10	.24
58	.02	.10	.18	-.12	.12	-.02	-.07	-.03	.04	-.00	.08
59	-.07	.47	.10	.08	.06	-.11	-.22	-.24	.08	.04	.37
60	-.09	.52	.11	-.01	.04	-.00	-.09	-.13	.02	.01	.31
61	-.07	.30	.11	-.16	.03	.04	-.12	-.13	.11	-.01	.18
62	-.04	.21	-.00	.02	-.04	-.02	-.06	-.15	-.07	.18	.11
63	-.20	.17	.06	-.06	.08	.06	-.05	-.16	-.03	.15	.14
64	-.07	.10	.08	-.33	-.12	-.11	-.06	-.05	.01	.23	.22

Table 2 (Continued)

Items	Factors										h^2
	I	II	III	IV	V	VI	VII	VIII	IX	X	
65	-.11	.01	.04	.01	.05	.15	-.07	-.21	.07	.31	.19
66	-.06	.21	.05	-.03	.02	.52	-.04	-.08	-.01	.04	.33
67	-.24	.19	.02	.06	.13	.10	-.09	-.23	.29	.19	.31
68	-.22	.24	.14	-.04	.01	.02	.02	-.30	.39	.18	.40
69	-.07	.04	-.01	-.07	.05	.00	-.02	.05	.05	.26	.09
70	-.24	.15	.02	-.13	-.07	.08	-.02	-.19	.03	.12	.16
71	-.06	.30	.07	-.01	.05	.14	.02	-.04	.13	.11	.15
72	-.02	-.00	.00	-.00	-.06	-.07	.12	.01	.07	.01	.03
73	.18	-.01	-.01	-.04	-.01	-.02	-.08	.00	-.02	-.02	.05
74	-.02	.07	.07	-.03	.12	.08	-.01	.00	-.05	.26	.11

were not taught to some of the pupils or that the concept was a difficult one for second grade pupils. This could also indicate that certain concepts had been mastered in grade one or had been taught in a relatively close period of time.

Factor I The items comprising this factor accounted for 2.57 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
7	-.41
52	-.36
57	-.36

Each of these items involves the identity element, zero, for addition and subtraction. Factor I was called the additive identity factor.

Factor II The items comprising this factor accounted for 4.09 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
60	+.52
59	+.47
41	+.41
10	+.39
54	+.37
56	+.37

Each of these items involved the fundamental operations of arithmetic, either individually or in combination. Factor II was called

the operations factor.

Factor III The items comprising this factor accounted for 2.86 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
3	+ .70
39	+ .70
38	+ .66
1	+ .43

Each of these items required that the pupil write a numeral after hearing the words for that numeral. Factor III was called the numeral from words factor.

Factor IV The items comprising this factor accounted for 2.00 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
40	- .50
8	- .40
42	- .35
64	- .33

Each of these items involves the ordinal number concept as well as a particular pattern. Factor IV was called the ordinal pattern factor.

Factor V The items comprising this factor accounted for 1.44 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
17	+ .61
44	+ .60

These items both involve ordering two numbers using the "less than" or the "greater than" relation. Factor V was called the order relation factor.

Factor VI Items comprising this factor accounted for 1.30 per cent of the variance. The items loaded on this factor were:

Item	Factor Loading
66	.52
14	.40

Each of these items involved the identification of the unit fractional part of a figure which was shaded. Factor VI was called the unit fractional part factor.

Factor VII The items comprising this factor accounted for 1.45 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
48	- .48
35	- .25

Two items loaded on this factor, with item fifty loading fairly high also. However, item fifty loaded higher on Factor VIII and will be included with items loading on that factor. Item thirty-five asks about a geometric concept while item forty-eight involves a basic addition fact. This factor was called a mathematical factor.

Factor VIII The items comprising this factor accounted for 4.59

per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
53	-.45
31	-.45
29	-.44
51	-.41
5	-.40
49	-.38
26	-.37
50	-.35
28	-.34
30	-.34
45	-.33
4	-.33
27	-.32

Each of these items involves either the addition or the subtraction of whole numbers. Factor VIII was called the addition and subtraction of whole numbers factor.

Factor IX The items comprising Factor IX accounted for 1.37 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
68	+.39
15	+.38
18	+.34

Each of the items loading on this factor involves writing the number representing a quantity of counting sticks. Factor IX was called a place value factor.

Factor X The items comprising this factor accounted for 2.10 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
13	+.36
65	+.31
9	+.27

Each of the items loading on this factor involves some number pattern; numerals before and after, story which goes with a number sentence, or the pattern of numbers missing from a table. Factor X was called a general number pattern factor.

Grade three factor analysis

The factor analysis for grade three is shown in Table 3. It indicates that the mathematical concepts developed in grade three are more complex and interrelated than those of the previous grades. Also, the diversity of the order in which topics are presented in various textbooks beginning to be evident.

There were ten factors which accounted for 20.77 per cent of the variance. The indicators of the test items, h^2 , in Table 3, demonstrates that the items are relatively inconsistent, ranging from .05 for item forty-four to .36 for item one.

Factor I The items comprising this factor accounted for 5.91 per cent of the variance. Items which loaded on this factor were:

Table 3. Rotated factor loadings for grade three

Items	Factors										h ²
	I	II	III	IV	V	VI	VII	VIII	IX	X	
1	-.45	.11	-.05	-.27	-.07	-.05	-.07	-.19	-.01	.11	.35
2	-.33	.14	.09	-.20	-.04	-.04	.10	-.00	.06	.04	.19
3	-.31	.14	.05	-.09	.04	.04	.14	-.20	-.04	.11	.20
4	-.51	.08	.07	-.11	.02	-.00	-.03	-.16	.01	-.08	.32
5	.02	-.05	.11	-.08	-.07	.14	-.12	.00	-.01	.09	.07
6	-.48	.05	.06	-.13	.03	-.03	.12	.12	-.08	.06	.30
7	-.25	.19	.06	-.10	.21	-.18	-.09	-.03	-.31	.11	.31
8	-.48	.18	-.01	-.15	.08	-.09	-.06	-.25	-.06	.01	.37
9	-.43	.10	.20	.00	-.03	-.17	.02	-.04	.07	.01	.28
10	-.24	.06	.02	-.28	.03	-.12	-.21	.01	-.09	.17	.24
11	-.33	.22	.14	-.18	.09	-.26	.05	.03	-.17	.16	.34
12	-.34	.02	-.01	.01	.07	-.16	-.07	.03	.08	.11	.17
13	-.38	.11	.19	.01	.15	-.04	-.06	-.11	.07	.15	.26
14	-.28	.36	.12	-.17	.00	-.19	-.03	-.04	.02	.11	.30
15	-.22	.27	-.06	-.15	.08	.00	.18	-.04	.04	.18	.22
16	-.22	.12	.02	-.32	.15	-.10	.02	-.05	.04	.09	.20

Table 3 (Continued)

Items	Factors										h ²
	I	II	III	IV	V	VI	VII	VIII	IX	X	
17	-.23	.10	.19	-.15	.20	-.08	-.13	-.14	-.07	-.03	.21
18	-.32	.20	.06	-.22	-.01	-.01	.03	.02	-.01	.22	.25
19	-.06	.33	.03	.07	-.10	-.08	-.12	-.13	-.03	.09	.17
20	-.20	.27	.20	-.05	.16	-.10	.06	-.10	.02	.10	.22
21	-.16	.15	.26	-.11	.05	-.17	-.00	-.07	.05	.18	.20
22	-.12	.14	.16	-.30	.09	-.05	-.06	.02	.12	-.12	.20
23	-.11	.18	-.00	-.14	.08	-.02	.01	-.03	-.02	.34	.19
24	-.34	.29	.23	-.16	.12	-.04	-.21	.02	-.03	-.02	.34
25	-.19	-.01	.24	.03	.13	-.11	.06	-.08	-.04	.05	.14
26	-.24	.06	.17	.03	.06	-.09	.08	.16	.06	.02	.14
27	-.36	.29	.04	-.03	.13	-.14	-.02	.05	.06	.14	.28
28	-.03	.09	.01	-.03	-.01	-.39	.04	-.15	-.02	.06	.19
29	-.14	.15	.33	-.15	-.09	-.03	.02	-.07	.05	.12	.21
30	-.26	.01	.02	-.12	-.02	-.24	.04	-.17	.09	.05	.18
31	-.20	.16	.02	-.19	.10	-.40	-.12	-.00	.08	.02	.29
32	-.04	.06	.03	-.02	.05	-.06	-.01	-.07	.04	.30	.11

Table 3 (Continued)

Items	Factors										h ²
	I	II	III	IV	V	VI	VII	VIII	IX	X	
33	-.13	.40	.04	-.08	.03	-.11	.03	-.08	.14	.06	.23
34	-.02	.22	.13	-.00	.15	-.15	.09	-.30	-.09	.10	.23
35	-.07	.21	-.02	-.09	.19	-.23	.05	.03	.20	.06	.20
36	-.19	.11	.26	-.25	.16	-.12	.06	-.06	-.02	.12	.24
37	.03	-.05	-.08	-.01	.38	-.02	.05	.02	-.03	.04	.16
38	-.09	-.00	.07	-.43	.05	-.08	.05	-.09	.07	.10	.23
39	-.09	.20	.12	-.01	.26	-.21	-.05	-.02	.03	-.03	.18
40	-.02	.01	.03	-.02	.09	-.04	.35	-.01	-.04	-.00	.14
41	-.16	.03	.06	-.16	.04	-.30	.06	.02	.12	.10	.18
42	-.19	-.02	.10	-.03	.09	-.03	.01	-.10	.26	.04	.14
43	-.06	.14	-.05	-.11	.07	-.08	.02	-.28	.06	.06	.13
44	.02	.05	.05	-.04	.03	-.06	-.03	-.00	.19	.02	.05
45	-.14	.02	.18	-.01	.12	-.05	-.05	-.24	.11	.11	.16
46	-.08	.12	-.02	-.10	.30	-.05	-.01	-.08	.14	.16	.18
47	-.03	.03	.05	-.05	.28	-.01	.07	-.07	.05	.02	.10
48	-.14	.27	.01	-.12	.11	-.03	.14	-.05	-.04	.05	.15

Item	Factor Loading
4	-.51
6	-.48
8	-.48
1	-.45
9	-.43

With the exception of item six, these items involve the notion of place value. All of these items were easy for the pupils to answer. Item eight was the most difficult, 67 per cent of the students answered it correctly while 88 per cent answered item six correctly. Difficulty of the items played a role in the loadings on this factor. Factor I was called the place value factor.

Factor II The items comprising this factor accounted for 2.82 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
33	+.40
14	+.36
19	+.33

Although each of these items seems to involve a different concept, each of them requires computation to arrive at the correct response. Factor II was called the computation factor.

Factor III The items comprising this factor accounted for 1.67 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
29	+ .33
21	+ .26
36	+ .26

Each of the items involves a property of the natural number system. Factor III was called the number property factor.

Factor IV The items comprising this factor accounted for 2.21 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
38	- .43
16	- .32
22	- .30

The items loading on this factor involve the completion of a number relationship. Factor IV was called a number relation factor.

Factor V The items comprising this factor accounted for 1.67 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
37	+ .38
46	+ .30
47	+ .28
39	+ .26

Three of these items, thirty-seven, thirty-nine, and forty-seven involve geometric concepts while item forty-six involves renaming necessitated by the need to "borrow" in subtraction. However, each of

them involves the "naming" concept. Factor V was called the naming factor.

Factor VI The items comprising this factor accounted for 2.02 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
31	-.40
28	-.39
41	-.30

Factor VI was called matching factor because each of the items loading on this factor involves matching a picture or word sentence to the corresponding mathematical statement.

Factor VII The items comprising this factor accounted for 0.92 per cent of the variance. Two items loaded on this factor which were unrelated as far as a common mathematical concept was concerned. One of these items loaded positively while the other loaded negatively. No name was given to this factor.

Item	Factor Loading
40	+.35
24	-.21

Factor VIII The items comprising this factor accounted for 1.26 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
34	-.30
43	-.28
45	-.24

Items loading on this factor involve different mathematical concepts: item thirty-four asks the pupil to convert cups or pints to quarts; item forty-three asks about symmetry; while item forty-five asks about place value in the vertical multiplication algorithm. All of the items were fairly difficult so this was primarily a difficulty factor.

Factor IX The items comprising this factor accounted for 0.94 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
42	+.26
7	-.31

Other than the fact that they both involve mathematical notions, there seems to be nothing in common. Item seven involves ratio and proportion while item forty-two asks for the number of straight lines which can be drawn between two points. Factor IX was called a mathematical factor.

Factor X The items comprising this factor accounted for 1.37 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
23	+.34
32	+.30

These items both involve identifying the fractional part of a picture which was shaded. Factor X was called the identifying fractional parts factor.

Grade four factor analysis

Because of the introduction of more complex mathematical notions along with the expectation of mastery and retention of previously learned concepts, the factor analysis indicated more advanced factors. Seven factors were shown on Table 4. These factors accounted for 14.83 per cent of the variance.

The indicators of the variance assumed by each of the test items, h^2 , in Table 4, demonstrates that the items were relatively consistent. The range was from .02 for forty-nine and fifty-one to .34 for item three.

Factor I The combination of items comprising this factor accounted for 2.66 per cent of the variance. The items which loaded on this factor were:

Table 4. Rotated factor loadings for grade four

Items	I	II	III	IV	Factors V	VI	VII	h^2
1	.01	-.10	-.02	.37	-.05	-.09	.06	.16
2	-.02	-.48	-.03	.26	-.06	-.02	.01	.30
3	-.12	-.54	-.11	.12	.08	-.03	-.00	.34
4	-.04	-.51	-.03	.21	-.06	-.04	.03	.31
5	-.10	-.08	.14	.37	-.05	-.14	.00	.20
6	-.04	-.10	-.19	.40	-.00	-.00	.14	.23
7	-.22	-.09	-.20	.14	.01	-.04	.29	.20
8	-.21	-.07	-.03	.18	-.21	-.05	-.15	.15
9	-.24	-.07	.02	.41	-.01	-.05	.01	.23
10	-.08	-.04	-.12	.46	-.09	-.02	-.07	.24
11	.05	-.05	-.03	.35	.05	.00	-.07	.14
12	-.11	.00	-.04	.21	-.21	-.06	-.21	.15
13	-.14	-.04	-.09	.34	-.14	-.10	-.03	.18
14	-.20	.01	.14	.40	-.13	.01	-.03	.24
15	-.22	-.02	.01	.32	-.12	-.20	-.01	.20
16	-.18	-.06	-.00	.26	.02	.02	-.24	.16

Table 4 (Continued)

Items	Factors							h ²
	I	II	III	IV	V	VI	VII	
17	-.16	-.05	.02	.25	-.23	-.11	.04	.16
18	-.20	-.05	.02	.34	-.09	-.01	.05	.17
19	-.11	-.12	-.17	.16	-.25	.06	-.16	.17
20	-.03	-.11	-.08	.21	-.14	-.39	-.13	.25
21	-.34	-.03	.06	.18	-.13	.09	.09	.18
22	-.14	-.06	-.08	.36	-.10	.02	-.13	.18
23	-.30	-.10	.00	-.00	-.10	-.08	-.02	.12
24	-.24	-.04	-.02	.20	-.07	-.13	-.06	.12
25	-.10	-.10	-.10	.13	-.06	-.46	-.11	.28
26	-.41	-.01	.05	.19	-.06	-.08	.02	.22
27	-.13	-.01	.11	.13	-.35	-.09	-.05	.18
28	-.10	-.07	.05	.09	-.04	-.06	-.02	.03
29	-.20	-.05	-.03	.20	-.02	-.13	-.09	.11
30	-.26	.04	-.08	.24	-.19	-.08	.03	.18
31	-.05	-.03	-.21	.17	.09	.08	-.26	.16
32	-.17	-.02	.03	.24	-.09	-.20	-.02	.14

Table 4 (Continued)

Items	Factors							h^2
	I	II	III	IV	V	VI	VII	
33	-.15	-.00	-.20	.09	-.02	-.07	-.01	.08
34	-.29	-.03	-.01	.18	-.17	-.10	-.04	.16
35	-.01	-.08	-.13	.21	-.07	-.01	-.06	.08
36	-.13	.02	-.03	-.01	-.35	-.07	-.03	.15
37	-.32	-.02	-.10	.13	-.09	-.06	.00	.14
38	-.07	.01	.05	-.00	-.02	-.17	.06	.04
39	.04	.04	.01	.06	-.33	-.05	.03	.12
40	-.06	.00	-.25	.07	-.21	-.01	.01	.12
41	-.03	.02	-.02	.00	-.24	-.05	-.04	.06
42	-.01	.16	-.19	.08	-.04	-.13	.09	.09
43	-.03	.01	-.06	.03	-.15	-.16	.11	.06
44	-.11	-.04	.00	.04	-.27	.02	-.04	.09
45	-.18	.05	-.01	.01	-.03	-.10	-.14	.07
46	-.17	-.10	-.11	-.10	-.08	-.02	.01	.07
47	-.04	-.03	-.13	.02	.04	.07	-.05	.02
48	-.08	.04	-.03	.02	-.09	-.14	.05	.04

Table 4 (Continued)

Items	Factors							h ²
	I	II	III	IV	V	VI	VII	
49	.03	.01	-.14	-.03	-.00	-.00	-.02	.02
50	-.07	-.05	.05	.04	-.12	-.03	.00	.03
51	.01	.00	.00	.00	.08	-.00	-.13	.02

Item	Factor Loading
26	-.41
21	-.34
37	-.32
23	-.30
34	-.29

With the exception of item twenty-three, these items all involve the multiplication operation. Item twenty-three requires the addition of one to find the whole numbers between 1647 and 1652. Because each item involves a property of the number system in addition to the operation, this factor was called the number property factor.

Factor II The combination of items comprising this factor accounted for 1.86 per cent of the variance. The items which loaded on this factor were:

Item	Factor Loading
3	-.54
4	-.51
2	-.48

These items were all ones which involved the basic facts for multiplication. They were facts which should have been committed to memory at least in the third grade. Factor II was called the basic multiplication factor.

Factor III The combination of items comprising this factor accounted for 1.19 per cent of the variance. The items which loaded

on this factor were:

Item	Factor Loading
40	-.25
33	-.20

Three other items, six, seven, and thirty-one loaded as highly on this factor as did these two. Because each of them loaded higher on another factor they were considered with that factor instead of with Factor III. The items which loaded on this factor involved two different mathematical concepts. Item thirty-three tested the idea the commutative property of multiplication while item forty asked the pupil to pick a parallelogram from among four geometric figures. They were relatively the same in difficulty. This factor was called a mathematical factor.

Factor IV The items comprising this factor accounted for 4.72 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
10	+.46
15	+.42
9	+.41
6	+.40
1	+.37
22	+.36
18	+.35
11	+.35
5	+.32

Each of the items loading on this factor involved a variety of the fundamental operations. Factor IV was called the multiple operations factor.

Factor V The combination of items comprising this factor accounted for 2.01 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
36	-.35
27	-.35
39	-.33

This combination of items involves factors of numbers, items twenty-seven and thirty-six, and the congruence relation between two angles. All three items were fairly difficult. This factor was called the factors of a number factor.

Factor VI The items comprising this factor accounted for 1.42 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
25	-.46
20	-.39

For each of these items, the pupil was to supply one of the digits in the result of addition, item twenty, and subtraction, item twenty-five. Factor VI was called the place value factor.

Factor VII The items comprising this factor accounted for 0.96 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
7	+.29
31	-.26
16	-.24
12	-.21

With the possible exception of item seven, all of these items involve some concept from geometry. Item seven asks about a pre, which is usually circular, so it can be said that there is a geometric concept to it. Factor VII was called the geometric factor.

Grade five factor analysis

Table 5 shows the ten factor analysis of the items on the test for grade five. As can be seen, h^2 , for each of the items is 1.00, indicating that the items were highly consistent. These ten factors account for 99.99 per cent of the variance.

Upon examining Table 5, it can be seen that quite a variety of items load highly on each of the factors and that several items load almost the same on several factors. An example is item twenty-one which loads +.54 on Factor II, -.56 on Factor IV, and -.54 on Factor VIII. These ten factors accounted for 99.99 per cent of the variance.

Factor I The items comprising this factor accounted for 11.96 per cent of the variance. Items which loaded on this factor were:

Table 5. Rotated factor loadings for grade five

Items	Factors										h ²
	I	II	III	IV	V	VI	VII	VIII	IX	X	
1	-.36	.16	-.01	-.34	.12	-.13	-.00	-.28	-.12	-.06	1.00
2	-.24	.59	-.05	-.68	.25	-.07	-.09	-.17	-.03	.16	1.00
3	-.48	.25	.13	-.23	-.67	.08	-.11	-.35	-.15	.15	1.00
4	-.22	.84	.26	-.12	.10	-.31	.03	-.11	-.20	-.10	1.00
5	-.04	.48	.21	-.67	-.05	-.25	-.09	-.29	-.33	-.00	1.00
6	-.14	.30	.77	-.23	-.02	.23	.06	-.33	-.27	-.04	1.00
7	-.22	.81	.01	-.38	-.17	-.07	-.08	-.26	-.18	-.12	1.00
8	-.87	.28	-.10	-.19	-.12	.10	.06	-.19	-.21	.08	1.00
9	-.29	.34	-.02	-.78	-.13	.14	.07	-.10	-.31	.21	1.00
10	-.25	.17	.02	-.32	-.78	.13	.09	-.26	-.27	-.17	1.00
11	-.17	.08	.55	-.53	-.45	.14	-.38	-.07	.10	-.02	1.00
12	-.32	.67	-.14	.11	-.24	-.18	.42	-.20	-.05	.32	1.00
13	-.51	.15	.10	-.51	-.32	.08	-.28	-.49	-.04	.15	1.00
14	-.40	.46	.28	-.12	-.26	.06	.15	-.36	-.55	.11	1.00
15	-.07	.70	.08	-.41	-.10	.23	-.14	-.33	-.36	-.09	1.00
16	.07	.89	-.12	-.29	-.17	.17	-.02	.09	-.02	-.22	1.00

Table 5 (Continued)

Items	Factors										h ²
	I	II	III	IV	V	VI	VII	VIII	IX	X	
17	-.46	.34	-.07	-.26	-.40	.29	-.05	-.38	-.46	-.04	1.00
18	-.53	.25	.20	-.41	-.31	-.07	-.36	-.13	-.28	.34	1.00
19	-.25	.17	.42	.08	-.12	.12	-.30	-.41	-.35	.56	1.00
20	-.13	.24	.00	-.22	-.13	-.02	-.08	.07	-.92	.05	1.00
21	.02	.54	-.08	-.56	-.15	.13	.20	-.54	-.04	-.14	1.00
22	-.44	.21	-.28	-.46	-.53	.10	.12	-.36	-.20	-.03	1.00
23	-.38	.54	.10	-.51	-.02	-.05	-.09	-.39	-.29	.20	1.00
24	-.41	.70	-.03	-.30	-.06	-.22	.04	-.08	-.28	.35	1.00
25	-.34	.22	-.05	-.15	-.08	-.05	.08	-.38	-.80	.06	1.00
26	-.06	.02	.04	-.33	-.41	.14	-.04	.01	-.22	-.26	1.00
27	-.49	.27	.22	-.46	-.27	-.20	-.08	-.22	-.48	.17	1.00
28	-.80	-.13	.36	-.23	-.14	.14	-.05	.23	-.08	-.16	1.00
29	-.56	.06	.38	-.47	.06	.31	.16	-.09	-.23	-.36	1.00
30	-.46	.17	.04	-.60	-.17	-.02	.30	-.49	-.15	-.15	1.00
31	-.02	.85	.23	-.03	-.28	.28	-.05	-.16	-.14	-.11	1.00
32	.12	.33	.11	-.70	-.09	.38	.30	.15	-.08	.32	1.00

Table 5 (Continued)

Items	Factors										h^2
	I	II	III	IV	V	VI	VII	VIII	IX	X	
33	.05	.30	.33	.05	-.15	.51	.20	-.47	-.45	-.20	1.00
34	.02	.27	.39	-.04	-.75	-.05	-.18	-.28	.23	.19	1.00
35	-.48	.28	.16	-.31	-.19	.02	-.19	-.28	-.64	.06	1.00
36	-.18	.25	.52	-.09	-.33	-.03	.62	-.14	-.35	.03	1.00
37	-.36	.38	.31	-.22	-.51	-.36	.38	.12	-.05	-.16	1.00
38	-.31	.34	-.26	-.42	-.46	-.12	-.19	-.52	.04	-.03	1.00
39	-.16	.03	.17	.06	-.38	-.02	-.85	.13	-.20	.12	1.00
40	-.05	.07	.11	-.21	.05	-.25	.30	-.87	-.06	-.16	1.00
41	-.13	.20	.15	-.12	-.13	.34	-.03	-.87	-.10	.07	1.00
42	-.10	.34	.10	.03	-.14	.09	-.01	-.22	.05	-.88	1.00
43	-.19	.22	.49	-.29	-.25	.02	.20	-.63	-.29	.02	1.00
44	.03	.36	.48	-.30	-.13	-.09	.03	-.19	-.55	-.42	1.00
45	-.77	.18	.07	-.09	.13	-.37	-.29	-.35	-.02	-.11	1.00
46	-.57	.50	.13	-.22	-.51	-.02	.06	-.23	-.17	-.09	1.00
47	-.21	.07	-.04	-.24	-.27	-.02	-.40	-.65	-.38	-.29	1.00
48	-.13	.56	.30	.00	-.36	-.14	.03	-.39	-.42	-.32	1.00

Table 5 (Continued)

Items	Factors										h^2
	I	II	III	IV	V	VI	VII	VIII	IX	X	
49	.06	-.03	.00	-.28	.03	.95	-.12	.01	.05	.04	1.00
50	.28	.10	.03	-.19	-.26	-.85	-.21	.13	-.09	.13	1.00
51	.14	-.21	-.04	-.07	-.31	.65	-.15	-.13	-.56	.25	1.00
52	-.46	.28	.00	.25	-.39	.29	.06	-.56	-.27	.11	1.00
53	.04	-.01	.01	.01	-.85	-.31	-.17	.02	-.38	-.06	1.00
54	-.29	-.25	.86	.24	-.09	-.19	-.05	.07	-.03	-.09	1.00
55	-.07	.47	-.22	.21	-.04	.62	-.43	.20	.07	-.27	1.00
56	.01	-.25	-.74	.09	-.12	.09	.08	.11	-.26	-.14	1.00

Item	Factor Loading
8	-.87
28	-.80
45	-.77

Factor II The items comprising this factor accounted for 15.22 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
16	+.89
31	+.85
4	+.84
7	+.81

Factor III The items comprising this factor accounted for 8.31 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
54	+.86
6	+.77

Factor IV The items comprising this factor accounted for 13.57 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
1	-.84
26	-.83
9	-.78

Factor V The items comprising this factor accounted for 10.46 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
53	-.85
10	-.78
34	-.75

Factor VI The items comprising this factor accounted for 7.95 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
49	+.95
50	+.85

Factor VII The items comprising this factor accounted for 5.76 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
36	+.62
39	-.85

Factor VIII The items comprising this factor accounted for 11.60 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
40	-.87
41	-.87

This factor involves the parallel and perpendicular relationship between two lines. Factor VIII was called the line relationship factor.

Factor IX The items comprising this factor accounted for 10.06 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
20	-.92
25	-.80

Item twenty asks the pupils to indicate the least common multiple of two numbers while item twenty-five asks for the greatest common factor. Each involves division so Factor IX was called the division factor.

Factor X The items comprising this factor accounted for 5.11 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
42	-.88
19	+.56

With the exception of the two factors which were named, the factors for grade five showed a mathematical relationship but no more specific notion than that. A common concept could not be identified for the items loading on Factors I, II, III, IV, V, VI, VII, or X.

Sixth grade factor analysis

The factors identified for the sixth grade showed, for the most part, the more advanced topics which are taught at this grade level. Such topics as integers, fractional and decimal operations, and rational number concepts were evident. There were seven factors as shown on Table 6. These seven factors accounted for 15.65 per cent of the variance.

The indicators of the variance assumed by each of the test items,

Table 6. Rotated factor loadings for grade six

Items	I	II	III	IV	Factors V	VI	VII	h^2
1	.08	-.12	-.09	.06	-.17	.06	-.10	.08
2	-.15	-.60	-.07	.02	-.17	.25	-.05	.48
3	-.08	-.02	-.20	-.10	-.00	.08	.03	.17
4	-.09	-.08	.25	.08	-.06	.27	-.10	.50
5	-.20	-.61	-.01	.06	-.18	.23	-.05	.08
6	-.06	-.06	-.25	.08	.02	.09	-.01	.08
7	-.06	-.20	.03	-.00	-.34	.04	-.11	.18
8	-.17	-.19	-.02	.02	-.06	.39	.01	.22
9	-.14	-.01	-.04	-.07	-.21	.41	-.02	.24
10	-.14	.03	-.06	.01	-.01	.01	-.20	.06
11	-.21	-.04	-.07	.05	.01	.07	-.07	.06
12	-.24	-.30	.04	-.11	-.33	.19	-.09	.31
13	-.09	-.01	.04	.04	-.26	.30	-.04	.17
14	-.26	-.27	.12	.08	-.26	.30	-.13	.33
15	-.11	-.11	.14	-.07	-.16	.25	-.06	.14
16	-.06	-.09	-.01	-.02	-.08	.08	-.45	.22

Table 6 (Continued)

Items	I	II	III	IV	Factors V	VI	VII	h^2
17	-.04	-.06	-.04	.04	-.06	.18	-.41	.21
18	-.00	-.09	-.11	-.22	-.15	.02	.05	.09
19	-.05	-.05	.23	-.03	-.24	.15	-.04	.14
20	-.03	-.13	.00	.23	-.11	-.04	-.07	.09
21	-.27	-.18	.09	-.08	-.14	.20	-.16	.21
22	-.18	-.01	-.02	.09	-.05	.12	.06	.06
23	-.02	-.06	-.02	.10	-.04	.37	-.09	.16
24	-.20	-.19	.19	.04	-.05	.29	-.00	.20
25	-.40	-.16	.23	-.04	-.13	.13	.02	.27
26	-.17	-.03	-.03	-.08	-.09	.38	-.15	.21
27	-.11	-.13	.03	-.08	-.06	.44	-.09	.24
28	-.17	-.03	.14	-.11	-.02	.38	-.11	.22
29	-.07	-.15	.03	-.08	-.41	.17	-.06	.24
30	-.01	.09	-.02	-.23	-.08	.06	-.06	.08
31	-.08	-.01	-.02	.31	-.14	.18	-.07	.16
32	-.01	-.11	.10	.07	.00	.09	-.10	.04

Table 6 (Continued)

Items	Factors							h^2
	I	II	III	IV	V	VI	VII	
33	.01	.01	-.04	.08	-.18	.19	-.01	.08
34	-.22	-.06	-.16	-.00	.02	-.00	-.08	.09
35	-.13	-.11	.03	.01	-.18	.16	-.00	.09
36	-.17	-.02	.07	.05	-.26	.08	-.10	.12
37	-.23	-.12	-.02	-.00	-.25	.05	-.12	.15
38	-.34	-.15	.09	-.02	-.20	.11	-.06	.20
39	-.10	-.08	-.01	-.27	-.08	-.10	.00	.10
40	-.05	.00	-.04	.00	-.28	.08	.04	.09
41	-.04	.01	.19	-.01	.04	.03	.09	.05
42	.00	-.02	.19	-.01	-.11	.07	.02	.06
43	-.41	.05	-.01	.10	-.15	.12	-.11	.23
44	-.21	-.01	-.14	-.09	-.07	.10	.05	.09
45	-.08	.01	-.06	.22	-.08	-.07	.03	.07
46	-.19	-.04	-.02	-.05	-.01	.14	.00	.06
47	-.01	.05	-.19	-.03	-.12	-.05	-.05	.06
48	-.20	-.05	-.03	.02	-.11	.23	.04	.11
49	-.24	-.06	.14	.03	.01	.16	-.07	.11

h^2 , in Table 6, demonstrates that the items were relatively inconsistent, ranging from .05 for items thirty-two and forty-one to .50 for item five.

Factor I The combination of items comprising this factor accounted for 2.77 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
43	-.41
25	-.40
38	-.34

Each of the items loading on this factor involved geometric notions. This factor was called the geometric factor.

Factor II The combination of items comprising this factor accounted for 2.57 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
5	-.61
2	-.60

Both of these items asked the student to select the decimal numeral which was equivalent to a given fraction numeral. Factor II was called the decimal equivalent.

Factor III The combination of items comprising this factor accounted for 1.28 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
4	+ .25
6	- .25

Both of these items asked the student to write a given numeral in a different form, one in expanded notation and the other in fractional form. Factor III was called the renaming factor.

Factor IV The combination of items comprising this factor accounted for 1.09 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
31	+ .31
39	- .27
20	+ .23
30	- .23
18	- .22
45	- .22

The items loading on this factor included both numerical and geometrical concepts. Some of the numerical factors had positive loadings and some had negative loadings. The same was true for the items involving geometrical notions. Factor IV was called a mathematical factor.

Factor V The combination of items comprising this factor accounted for 2.62 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
29	-.41
7	-.34
12	-.33

These items all involve the concept of positive and negative numbers. Factor V was called the integer factor.

Factor VI The combination of items comprising this factor accounted for 3.95 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
27	+.44
9	+.41
8	+.39
28	+.38
23	+.37
26	+.37

Factor VI was named the rational number operations factors.

Factor VII The combination of items comprising this factor accounted for 1.38 per cent of the variance. Items which loaded on this factor were:

Item	Factor Loading
16	-.45
17	-.41

Factor VII was called the multiplication of fractions factor.

Analysis of Variance

In this section of Chapter IV, the results of the tests will be analyzed to determine the effect upon the scores which the (1) size of the city or town, (2) amount of money a district spends per pupil in the elementary school, (3) number of pupils in the elementary schools of a district, (4) the valuation per pupil in the elementary schools in the district, (5) the geographic location of the school district in the State of Wisconsin, and (6) the sex of the pupils. The information for the first four of the above categories was supplied by the Wisconsin Department of Public Instruction and gives the 1968-1969 figures.

For purposes of this study, the data were analyzed using the one-way analysis of variance technique. The computed F-ratio was compared with the F-ratio at the .05 level and .01 level of significance for the indicated degree of freedom. If the computed F-ratio was found to be significant, the difference between pairs of treatment means was tested for significance. The t-test procedure described by Snedecor and Cochran (39) was used. This calculates t according to the following formula:

$$t_{\bar{X}_i - \bar{X}_j} = \frac{\bar{X}_i - \bar{X}_j}{s_{\bar{X}_i - \bar{X}_j}} \quad \text{with}$$

$$s_{\bar{X}_i - \bar{X}_j} = \sqrt{\frac{S_i^2}{n_i} + \frac{S_j^2}{n_j}}$$

where \bar{X}_i is the mean of the i^{th} treatment
 \bar{X}_j is the mean of the j^{th} treatment
 S_i is the variance of the i^{th} treatment
 S_j is the variance of the j^{th} treatment
 n_i is the number of scores in the i^{th} treatment
 n_j is the number of scores in the j^{th} treatment

The hypotheses were tested at the .05 level. If the F-ratio or t-test was significant at the .01 level, two asterisks were used. If significant at the .05 level, one asterisk was used.

For purposes of this study, the school districts in Wisconsin were separated into four categories, based upon the population within the boundaries of that district. When referring to a particular district the name of the city or town in which the district offices were located was used. Category A included those districts with a population of less than five thousand people. These schools were referred to as the small schools. Category B included those districts with a population greater than five thousand but less than twenty thousand people. The schools in Category B were called the medium schools. The third category, Category C, included those districts with a population greater than twenty thousand but less than two hundred thousand residents. The schools in Category C was referred to as large schools. Category D included those districts having a population greater than two hundred thousand. Schools in this category were referred to as extra-large districts.

Does the amount of money spent on each elementary school child have any effect on the ELMR test Score? To answer this question, the schools involved in this study were classified into three categories; these schools spending over \$800 per pupil, those spending between \$600 and \$800, and those schools spending less than \$600. The first category was indicated by A and was called those schools with a high expenditure. The second category, P, includes those schools with a medium expenditure. The third category, C, was made up of those schools whose expenditure was low.

Does the total enrollment in the elementary schools of a district influence the ELMR test scores? For purpose of this analysis, the districts were separated into large, medium, or small enrollments. Large schools, L, were schools with an elementary enrollment of more than five thousand pupils. Medium schools, M, had an enrollment of less than five thousand but more than one thousand five hundred. Schools with an enrollment of less than one thousand five hundred were called the small schools, S.

Does the valuation per pupil in the district influence the ELMR test scores? To complete this analysis the school districts were identified as being in one of three categories, high, medium, or low. A high valuation, H, was greater than \$34,000. A medium valuation, M, is less than \$34,000 but greater than \$20,000. A low valuation, L, is less than \$20,000.

Does the geographic location within the State of Wisconsin influence the ELMR test scores? The population of Wisconsin is

concentrated in the south-eastern part of the state. The school districts were identified as being in the northern, N, portion of the state, central, C, or southern, S.

Is there a difference between the scores for boys and girls on the ELMR tests? The data were separated into the two categories and analyzed.

Grade one analysis of variance

The results of the analysis of variance for the six different null hypotheses for grade one are presented in this section.

Size of city or town

Ho: There is no significant difference between group mean test scores for Grade One as measured by the ELMR-1 when classified according to the size of the city or town in which the pupils attend school.

Table 7. Summary of analysis of variance for size of city or town for grade one

Source	SS	df	MS	F
Treatments	15660	3	5220	
Within Treatments	72450	842	86.04	60.66 **
Total	88110	845		

$F_{.01}(3, 842) = 3.80$ $F_{.05}(3, 842) = 2.61$

Examination of Table 7 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of means was analyzed using t-tests. Table 8 shows the information needed to complete this

analysis of the following null hypothesis:

Ho: There is no significant difference between the following pairs of mean test scores as measured by ELMR-1 when classified according to the size of the city or town in which the pupils attend school:

- 1) A vs B
- 2) A vs C
- 3) A vs D
- 4) B vs C
- 5) B vs D
- 6) C vs D

Table 8. Summary of tests for differences between pairs of group mean test scores for size of city or town for grade one

Group	N	Mean	Variance
A	135	46.30	58.33
B	184	48.71	84.76
C	343	48.16	62.06
D	184	37.67	150.52
Total	846	45.70	104.15

Because of the number of students involved, the critical value of t is $t_{.05^\infty} = 1.96$.

Using the formula for calculating t discussed earlier in this chapter, the following were calculated:

$$1) t_{\frac{\bar{X}_B - \bar{X}_A}{\sqrt{\frac{S^2}{N}}}} = 2.56^*$$

$$2) t_{\frac{\bar{X}_C - \bar{X}_A}{\sqrt{\frac{S^2}{N}}}} = 2.38^*$$

$$3) \quad t_{\bar{X}_A - \bar{X}_D} = 7.72^{**}$$

$$4) \quad t_{\bar{X}_B - \bar{X}_C} = 0.68$$

$$5) \quad t_{\bar{X}_B - \bar{X}_D} = 9.77^{**}$$

$$6) \quad t_{\bar{X}_C - \bar{X}_D} = 10.50^{**}$$

Based upon the above, null hypothesis 1, 2, 3, 5, 6 were rejected, 4 was accepted. There is a significant difference in each of these cases.

Per pupil expenditure

Ho: There is no significant difference between group mean test scores for grade one as measured by the ELMR-1 when classified by the expenditure per pupil for elementary education in the school district.

Table 9. Summary of analysis of variance for per pupil expenditure for grade one

Source	SS	df	MS	F
Treatments	2	6998	3499	
Within Treatments	843	81112	96.22	36.36**
Total	845	88110		

$F_{.01}(2, 843) = 4.64$ $F_{.05}(2, 843) = 3.01$

Examination of Table 9 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of means were analyzed using t-tests. Table 10 shows the information needed to complete the

analysis of the following null hypothesis:

Ho: There is no significant difference between the following pairs of mean test scores as measured by ELMR-1 when classified according to the expenditure per pupil for elementary school education in the school district:

1) H vs M

2) H vs L

3) M vs L

Table 10. Summary of tests for differences between pairs of group mean test scores for per pupil expenditure for grade one.

Group	N	Mean	Variance
H	169	48.96	75.38
M	468	43.12	124.34
L	209	48.85	48.72
Total	846	45.70	104.15

$$t_{.05^{\infty}} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were calculated:

$$1) \quad t_{\frac{\bar{X}_H - \bar{X}_M}{\sqrt{\frac{S_H^2}{n_H} + \frac{S_M^2}{n_M}}}} = 6.93^{**}$$

$$2) \quad t_{\frac{\bar{X}_H - \bar{X}_L}{\sqrt{\frac{S_H^2}{n_H} + \frac{S_L^2}{n_L}}}} = 0.13$$

$$3) \quad t_{\frac{\bar{X}_L - \bar{X}_M}{\sqrt{\frac{S_L^2}{n_L} + \frac{S_M^2}{n_M}}}} = 8.13^{**}$$

Based on the above, null hypothesis 1 and 3 were rejected, while 2 was accepted. There is a significant difference between the mean

scores of schools whose per pupil expenditure is high and those whose expenditure is medium. There is also a significant difference between the mean scores of schools whose per pupil expenditure is medium and those whose per pupil expenditure is low.

Enrollment in elementary schools

Ho: There is no significant difference between group mean test scores for Grade One as measured by ELMR-1 when classified according to the enrollment in the elementary schools of the school district.

Table 11. Summary of the analysis of variance for enrollment in elementary schools for grade one

Source	SS	df	MS	F
Treatments	2482	2	1241	
Within Treatments	85630	843	101.6	12.22**
Total	88112	845		

$F_{.01}(2, 843) = 4.64$ $F_{.05}(2, 843) = 3.01$

Examination of Table 11 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of means were analyzed using t-tests. Table 12 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-1 when classified according to the enrollment in the elementary school of the school district:

- 1) L vs M
- 2) L vs S
- 3) M vs S

Table 12. Summary of tests for differences between pairs of group mean test scores for enrollment in elementary schools for grade one

Group	N	Mean	Variance
L	527	44.50	117.94
M	184	48.71	84.76
S	135	46.30	58.33
Total	846	45.70	104.15

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were calculated:

$$1) t_{\frac{\bar{X}_M - \bar{X}_L}{\sqrt{\frac{117.94 + 84.76}{2}}}} = 40.12^{**}$$

$$2) t_{\frac{\bar{X}_S - \bar{X}_L}{\sqrt{\frac{58.33 + 117.94}{2}}}} = 73.69^{**}$$

$$3) t_{\frac{\bar{X}_M - \bar{X}_S}{\sqrt{\frac{84.76 + 58.33}{2}}}} = 28.00^{**}$$

Based on the above, all three of the null hypotheses were rejected. There is a highly significant difference between the mean scores of schools with respect to the enrollment in the elementary schools of a district.

Per pupil valuation of the school district

Ho: There is no significant difference between group mean test scores for Grade One as measured by ELMR-1 when classified according to the valuation per pupil of the school district.

Table 13. Summary of the analysis of variance for per pupil valuation for grade one

Source	SS	df	MS	F
Treatments	4048	2	2024	
Within Treatments	84062	843	99.72	20.30**
Total	84110	845		

$$F_{.01}(2, 843) = 4.64 \quad F_{.05}(2, 843) = 3.01$$

Examination of Table 13 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of means were analyzed using t-tests. Table 14 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-1 when classified according to the valuation per pupil for grade one:

- 1) H vs M
- 2) H vs L
- 3) M vs L

Table 14. Summary of tests for differences between pairs of group mean test scores for per pupil valuation for grade one

Group	N	Mean	Variance
H	455	44.02	134.99
M	250	49.01	56.49
L	141	45.26	60.42
Total	846	45.70	104.15

$$t_{.05^{\infty}} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were calculated:

$$1) \quad t_{\frac{\bar{X}_M - \bar{X}_H}{\sqrt{\frac{134.99 + 56.49}{2}}}} = 6.90^{**}$$

$$2) \quad t_{\frac{\bar{X}_L - \bar{X}_H}{\sqrt{\frac{60.42 + 56.49}{2}}}} = 1.45$$

$$3) \quad t_{\frac{\bar{X}_M - \bar{X}_L}{\sqrt{\frac{134.99 + 60.42}{2}}}} = 4.64^{**}$$

Based on the above, null hypotheses one and three were rejected while two was accepted. There is a significant difference between the mean scores of high and medium valuation per pupil and between schools of low and medium valuation per pupil. There is no significant difference between schools of high and low valuation per pupil.

Geographic location of the school district

Ho: There is no significant difference between group mean test scores for Grade One as measured by the ELMR-1 when classified according to the geographic location of the school district.

Table 15. Summary of the analysis of variance for geographic location for grade one

Source	SS	df	MS	F
Treatments	956	2	478	
Within Treatments	87154	843	103.4	4.62*
Total	88110	845		

$F_{.01}(2, 843) = 4.64$ $F_{.05}(2, 843) = 3.01$

Examination of Table 15 and the critical F-ratio indicates that the F-ratio of 4.62 is significant at the .05 level of significance, but not at the .01 level. Because of this, differences between pairs of means were analyzed using t-tests. Table 16 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of mean test scores as measured by ELMR-1 when classified according to the geographic location of the school district:

- 1) N vs C
- 2) N vs S
- 3) C vs S

Table 16. Summary of tests for differences between pairs of group mean test scores for geographic location for grade one

Group	N	Mean	Variance
N	140	44.71	77.92
C	227	47.39	61.67
S	479	44.90	129.95
Total	846	45.70	104.15

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were calculated:

$$1) \quad t_{\frac{\bar{X}_C - \bar{X}_N}{\sqrt{\frac{S^2}{n}}}} = 1.85$$

$$2) \quad t_{\frac{\bar{X}_S - \bar{X}_N}{\sqrt{\frac{S^2}{n}}}} = 0.89$$

$$3) \quad t_{\frac{\bar{X}_C - \bar{X}_S}{\sqrt{\frac{S^2}{n}}}} = 3.38^*$$

Based on the above, null hypothesis three was rejected and null hypotheses one and two were accepted. There is a significant difference between mean group test scores of school districts in central Wisconsin and those in southern Wisconsin.

Sex

Ho: There is no significant difference between group mean test scores for Grade One as measured by ELMR-1 when classified according to sex.

Table 17. Summary of the analysis of variance for sex for grade one

Source	SS	df	MS	F
Treatments	119.8	1	119.8	
Within Treatments	87991	844	104.3	1.149
Total	88110	845		

$F_{.01}(1, 844) = 6.68$ $F_{.05}(1, 844) = 3.85$

Examination of Table 17 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference

between the group mean test score for boys and girls.

Grade two analysis of variance

The results of the analysis of variance for the six different null hypotheses for grade two are presented in this section.

Size of city or town

Ho: There is no significant difference between group mean test scores for Grade Two as measured by the ELMR-2 when classified according to the size of the city or town in which the pupils attend school.

Table 18. Summary of the analysis of variance for size of city or town for grade two

Source	SS	df	MS	F
Treatments	16820	3	5607	
Within Treatments	97850	802	122	45.95**
Total	114700	805		

$F_{.01}(3, 802) = 3.81$	$F_{.05}(3, 802) = 2.62$
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Examination of Table 18 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of means were analyzed using t-tests. Table 19 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-2 when classified according to the size of the city or town in which the pupils attend school:

- 1) A vs B
- 2) A vs C
- 3) A vs D
- 4) B vs C
- 5) B vs D
- 6) C vs D

Table 19. Summary of tests for differences between pairs of group mean test scores for size of city or town for grade two

Group	N	Mean	Variance
A	201	50.36	127.77
B	146	55.47	119.85
C	360	51.84	100.68
D	99	39.23	186.14
Total	806	50.58	142.27

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in the chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_B - \bar{X}_A}{\sqrt{\frac{1}{N_B} + \frac{1}{N_A}}}} = 4.24^*$$

$$2) \quad t_{\frac{\bar{X}_C - \bar{X}_A}{\sqrt{\frac{1}{N_C} + \frac{1}{N_A}}}} = 1.55$$

$$3) \quad t_{\frac{\bar{X}_A - \bar{X}_D}{\sqrt{\frac{1}{N_A} + \frac{1}{N_D}}}} = 7.02^{**}$$

$$4) \quad t_{\frac{\bar{X}_B - \bar{X}_C}{\sqrt{\frac{1}{N_B} + \frac{1}{N_C}}}} = 3.45^*$$

$$5) \quad t_{\bar{X}_B - \bar{X}_D} = 9.88^{**}$$

$$6) \quad t_{\bar{X}_C - \bar{X}_D} = 8.59^{**}$$

Based on the above, null hypothesis two was accepted while all of the others were rejected. There is a significant difference between group mean test scores of small and medium sized cities, small and extra large cities, medium and large cities, medium and extra large cities, and large and extra large cities.

Per pupil expenditure

Ho: There is no significant difference between group mean test scores for Grade Two as measured by ELMR-2 when classified according to the expenditure per pupil for elementary school education in the school district.

Table 20. Summary of analysis of variance for per pupil expenditure for grade two

Source	SS	df	MS	F
Treatments	6764	2	3382	
Within Treatments	107936	803	134.4	25.17**
Total	114700	805		

$F_{.01}(2, 803) = 4.64$ $F_{.05}(2, 803) = 3.01$

Examination of Table 20 and the critical F-ratio led to the rejection of the null hypothesis. Because of this, differences between pairs of means were analyzed using t-tests. Table 21 shows the information needed to complete the analysis of the following null hypothesis:

Ho: There is no significant difference between the following pairs of mean test scores, as measured by ELMR-2 when classified according to expenditure per pupil for elementary school education in the school districts:

1) H vs M

2) H vs L

3) M vs L

Table 21. Summary of tests for differences between pairs of group mean test scores for per pupil expenditure for grade two

Group	N	Mean	Variance
H	159	55.32	126.02
M	437	48.11	155.76
L	210	52.14	94.30
Total	806	50.58	142.27

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_H - \bar{X}_M}{\sqrt{\frac{1}{N_H} + \frac{1}{N_M}}}} = 6.73^{**}$$

$$2) \quad t_{\frac{\bar{X}_H - \bar{X}_L}{\sqrt{\frac{1}{N_H} + \frac{1}{N_L}}}} = 2.85^*$$

$$3) \quad t_{\frac{\bar{X}_L - \bar{X}_M}{\sqrt{\frac{1}{N_L} + \frac{1}{N_M}}}} = 4.50^*$$

Based on the above, all three of the null hypotheses were rejected.

The differences between the means is significant.

Enrollment in elementary schools

Ho: There is no significant difference between group mean test scores for Grade Two as measured by ELMR-2 when classified according to the enrollment in the elementary schools of the school district.

Table 22. Summary of the analysis of variance for enrollment in elementary schools for grade two

Source	SS	df	MS	F
Treatments	4467	2	2234	
Within Treatments	110233	803	137.2	16.28**
Total	114700	805		

$F_{.01}(2, 803) = 4.64$ $F_{.05}(2, 803) = 3.01$

Examination of Table 22 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of group means were analyzed using t-tests. Table 23 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-2 when classified according to the enrollment in elementary schools for Grade Two:

- 1) L vs M
- 2) L vs S
- 3) M vs S

Table 23. Summary of tests for differences between pairs of group mean test scores for enrollment in elementary schools for grade two

Group	N	Mean	Variance
L	459	49.12	146.02
M	146	55.47	119.85
S	201	50.36	127.77
Total	806	50.58	142.27

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) t_{\bar{X}_M - \bar{X}_L} = 5.94^*$$

$$2) t_{\bar{X}_S - \bar{X}_L} = 1.26$$

$$3) t_{\bar{X}_M - \bar{X}_S} = 5.11^*$$

Based on the above, null hypothesis two was accepted, but null hypotheses one and three were rejected. There was a significant difference between the group mean test scores for large and medium elementary school enrollments and between medium and small elementary school enrollments. There was no significant difference between the group mean test scores for schools with large and small elementary school enrollments.

Per pupil valuation of the school district

Ho: There is no significant difference between group mean test scores for Grade Two as measured by ELMR-2 when classified according to the valuation per pupil of the school district.

Table 24. Summary of the analysis of variance for per pupil valuation for grade two

Source	SS	df	MS	F
Treatments	1629	2	814.5	
Within Treatments	113071	803	140.8	5.79**
Total	114700	805		

$F_{.01}(2, 803) = 4.64$ $F_{.05}(2, 803) = 3.01$

Examination of Table 24 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of group means were analyzed using t-tests. Table 25 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-2 when classified according to the valuation per pupil of the school district:

- 1) H vs M
- 2) H vs L
- 3) M vs L

Table 25. Summary of tests for differences between pairs of group mean test scores for per pupil valuation for grade two

Group	N	Mean	Variance
H	513	49.57	163.68
M	190	52.95	107.13
L	103	51.21	84.63
Total	806	50.58	142.27

$$t_{.05^{\infty}} = 1.96$$

Using the formula for calculating t discussed earlier in the chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_M - \bar{X}_H}{\sqrt{\frac{163.68 + 107.13}{2}}}} = 3.60^*$$

$$2) \quad t_{\frac{\bar{X}_L - \bar{X}_H}{\sqrt{\frac{163.68 + 84.63}{2}}}} = 1.53$$

$$3) \quad t_{\frac{\bar{X}_M - \bar{X}_L}{\sqrt{\frac{107.13 + 84.63}{2}}}} = 1.48$$

Based on the above, the first null hypothesis was the only one which was rejected. The difference between the mean of schools with a high valuation per pupil and the mean of schools with a medium valuation per pupil is significant at the .05 level.

Geographic location

Ho: There is no significant difference between group mean test scores for Grade Two as measured by ELMR-2 when classified according to the geographic location of the school district.

Table 26. Summary of analysis of variance for geographic location for grade two

Source	SS	df	MS	F
Treatments	1157	2	578.5	
Within Treatments	113543	803	141.4	4.092*
Total	114700	805		

$F_{.01}(2, 803) = 4.64$ $F_{.05}(2, 803) = 3.01$

Examination of Table 26 and the critical F-ratio indicates that the F-ratio of 4.62 is significant at the .05 level of significance, but not at the .01 level. Because of this, difference between pairs of means were analyzed using t-tests. Table 27 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-2 when classified according to the geographic location of the school district:

- 1) N vs C
- 2) N vs S
- 3) C vs S

Table 27. Summary of tests for differences between pairs of group mean test scores for geographic location for grade two

Group	N	Mean	Variance
N	65	53.65	67.74
C	218	49.05	95.51
S	523	50.84	168.82
Total	806	50.58	142.27

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in the chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_N - \bar{X}_C}{\sqrt{\frac{S^2}{n}}}} = 3.78^*$$

$$2) \quad t_{\frac{\bar{X}_N - \bar{X}_S}{\sqrt{\frac{S^2}{n}}}} = 2.04^*$$

$$3) \quad t_{\frac{\bar{X}_S - \bar{X}_C}{\sqrt{\frac{S^2}{n}}}} = 2.40^*$$

Based on the above, all of the null hypotheses were rejected. There is a significant difference between group mean test scores for all pairs in the geographic location categories.

Sex

Ho: There is no significant difference between group mean test scores for Grade Two as measured by ELMR-2 when classified according to sex.

Table 28. Summary of the analysis of variance for sex for grade two

Source	SS	df	MS	F
Treatments	8.188	1	8.188	
Within Treatments	114700	804	142.6	0.057
Total	114708	805		

$F_{.01}(1, 804) = 6.66$	$F_{.05}(1, 804) = 3.85$
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Examination of Table 28 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference between the group mean test score for boys and girls.

Grade three analysis of variance

The results of the analysis of variance for the six different null hypotheses for grade three are presented in this section.

Size of city or town

Ho: There is no significant difference between group mean test scores for Grade Three as measured by the ELMR-3 when classified according to the size of the city or town in which the pupils attend school.

Table 29. Summary of the analysis of variance for size of city or town for grade three

Source	SS	df	MS	F
Treatments	1588	3	529.3	
Within Treatments	54962	899	61.14	8.66**
Total	56550	902		

$F_{.01}(3, 899) = 3.81$ $F_{.05}(3, 899) = 2.61$

Examination of Table 29 and the critical F-ratios led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of means were analyzed using t-tests. Table 30 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-3 when classified according to the size of the city or town in which the pupils attend school:

- 1) A vs B
- 2) A vs C
- 3) A vs D
- 4) B vs C
- 5) B vs D
- 6) C vs D

Table 30. Summary of tests for differences between pairs of group mean test scores for size of city or town for grade three

Group	N	Mean	Variance
A	167	25.54	49.12
B	102	27.27	58.69
C	447	24.59	60.69
D	187	22.65	72.97
Total	903	24.67	62.62

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

- 1) $t_{\bar{X}_B - \bar{X}_A} = 1.86$
- 2) $t_{\bar{X}_A - \bar{X}_C} = 1.45$
- 3) $t_{\bar{X}_A - \bar{X}_D} = 3.50^*$

$$4) \quad t_{\frac{\bar{X}_B - \bar{X}_C}{\sqrt{\frac{SS}{df}}}} = 3.18^*$$

$$5) \quad t_{\frac{\bar{X}_B - \bar{X}_D}{\sqrt{\frac{SS}{df}}}} = 4.71^*$$

$$6) \quad t_{\frac{\bar{X}_C - \bar{X}_D}{\sqrt{\frac{SS}{df}}}} = 2.70^*$$

Based on the above, null hypotheses one and two were accepted while three, four, five, and six were rejected. There is no significant difference between group mean test scores between small and medium sized cities and between small and large sized cities. However, there is a significant difference between small and extra large cities, between medium and large cities, between medium and extra-large cities, and between large and extra-large cities.

Per pupil expenditure

Ho: There is no significant difference between group mean test scores for Grade Three as measured by ELMR-3 when classified according to the expenditure per pupil for elementary school education in the school district.

Table 31. Summary of analysis of variance for per pupil expenditure for grade three

Source	SS	df	MS	F
Treatments	1401	2	700.5	
Within Treatments	55150	900	61.28	11.43**
Total	56551	902		

$F_{.01}(2, 900) = 4.64$ $F_{.05}(2, 900) = 3.01$

Examination of Table 31 and the critical F-ratio led to the rejection of the null hypothesis. Because of this, differences between pairs of means were analyzed using t-tests. Table 32 shows the information needed to complete the analysis of the following null hypothesis:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-3 when classified according to expenditure per pupil for elementary school education in the school districts:

- 1) H vs M
- 2) H vs L
- 3) M vs L

Table 32. Summary of tests for differences between pairs of group mean test scores for per pupil expenditure for grade three

Group	N	Mean	Variance
H	197	27.01	65.57
M	403	24.16	58.79
L	303	23.82	61.18
Total	903	24.67	62.62

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\bar{X}_H - \bar{X}_M} = 4.12^*$$

$$2) \quad t_{\bar{X}_H - \bar{X}_L} = 4.36^*$$

$$3) \quad t_{\bar{X}_M - \bar{X}_L} = 0.58$$

Based on the above, null hypotheses one and two were rejected while three was accepted. There is a significant difference between group mean test scores because of per pupil expenditure for elementary school education between those schools whose expenditure is high and those schools whose expenditure is either medium or low. However, there is no significant difference between group mean test scores between schools whose per pupil expenditure is medium and those in which it is low.

Enrollment in elementary schools

Ho: There is no significant difference between group mean test scores for Grade Three as measured by ELMR-3 when classified according to the enrollment in the elementary schools of the district.

Table 33. Summary of the analysis of variance for enrollment in elementary schools for grade three

Source	SS	df	MS	F
Treatments	1066	2	532.8	
Within Treatments	55484	900	61.65	8.64*
Total	56550	902		

$F_{.01}(2, 900) = 4.64$ $F_{.05}(2, 900) = 3.01$

Examination of Table 33 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of group means were analyzed using t-tests. Table 34 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-3 when classified according to the enrollment in elementary schools for Grade Three:

- 1) L vs M
- 2) L vs S
- 3) M vs S

Table 34. Summary of tests for differences between pairs of group mean test scores for enrollment in elementary schools for grade three

Group	N	Mean	Variance
L	657	24.06	64.84
M	102	27.27	58.69
S	144	25.61	47.89
Total	903	24.67	62.62

$$t_{.05^{\infty}} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_M - \bar{X}_L}{\sqrt{\frac{s^2}{n_M} + \frac{s^2}{n_L}}}} = 3.92^*$$

$$2) \quad t_{\frac{\bar{X}_S - \bar{X}_L}{\sqrt{\frac{S^2}{n}}}} = 2.37^*$$

$$3) \quad t_{\frac{\bar{X}_M - \bar{X}_S}{\sqrt{\frac{S^2}{n}}}} = 1.75$$

Based on the above, null hypothesis one and two were rejected, while three was accepted. There is a significant difference between group mean test scores of schools with a large elementary school enrollment and those with either medium or small enrollments. There is no significant difference between group mean test scores of schools with medium and small enrollments.

Per pupil valuation of the school district

Ho: There is no significant difference between group mean test scores for Grade Three as measured by ELMR-3 when classified according to the valuation per pupil of the school district.

Table 35. Summary of the analysis of variance for the per pupil valuation for grade three

Source	SS	df	MS	F
Treatments	30.56	2	15.28	
Within Treatments	56520	900	62.50	0.24
Total	56550	902		

$F_{.01}(2, 900) = 4.64$ $F_{.05}(2, 900) = 3.01$

Examination of Table 35 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference between group mean test scores because of valuation per pupil in the school district.

Geographic location

Ho: There is no significant difference between group mean test scores for Grade Three as measured by ELMR-3 when classified according to the geographic location of the school district.

Table 36. Summary of the analysis of variance for geographic location for grade three

Source	SS	df	MS	F
Treatments	10.56	2	5.28	
Within Treatments	56540	900	62.82	0.084
Total	56550	902		

$F_{.01}(2, 900) = 4.64$ $F_{.05}(2, 900) = 3.01$

Examination of Table 36 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference between group mean test scores because of the geographic location of the school district.

Sex

Ho: There is no significant difference between group mean test scores for Grade Three as measured by ELMR-3 when classified according to sex.

Table 37. Summary of the analysis of variance for sex for grade three

Source	SS	df	MS	F
Treatments	3.81	1	3.81	
Within Treatments	56550	901	62.76	0.061
Total	56553	902		

$F_{.01}(1, 901) = 6.67$ $F_{.05}(1, 901) = 3.85$

Examination of Table 37 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference between the group mean test score for boys and girls.

Grade four analysis of variance

The result of the analysis of variance for the six different null hypotheses for grade four are presented in this section.

Size of city or town

Ho: There is no significant difference between group mean test scores for Grade Four as measured by the ELMR-4 when classified according to the size of the city or town in which the pupils attend school.

Table 38. Summary of analysis of variance for size of city or town for grade four

Source	SS	df	MS	F
Treatments	2308	3	769.3	
Within Treatments	38402	890	43.15	17.83**
Total	40710	893		

$F_{.01}(3, 890) = 3.81$ $F_{.05}(3, 890) = 2.61$

Examination of Table 38 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of group mean test scores were analyzed using t-tests. Table 39 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-4 when classified according to the size of city or town in which the pupil attend schools:

- 1) A vs B
- 2) A vs C
- 3) A vs D
- 4) B vs C
- 5) B vs D
- 6) C vs D

Table 39. Summary of tests for difference between pairs of group mean test scores for size of city or town for grade four

Group	N	Mean	Variance
A	189	24.75	42.01
B	83	25.61	32.86
C	433	26.22	44.80
D	189	22.07	44.12
Total	894	24.97	45.54

$$t_{.05^{\infty}} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\bar{X}_B - \bar{X}_A} = 1.10$$

$$2) \quad t_{\bar{X}_C - \bar{X}_A} = 2.57^*$$

$$3) \quad t_{\bar{X}_A - \bar{X}_D} = 3.97^*$$

$$4) \quad t_{\bar{X}_C - \bar{X}_B} = 0.85$$

$$5) \quad t_{\bar{X}_B - \bar{X}_D} = 4.74^*$$

$$6) \quad t_{\bar{X}_C - \bar{X}_D} = 7.15^*$$

Based on the above, null hypotheses one and four were accepted while two, three, five, and six were rejected. There is no significant difference between group mean test scores between small and medium sized cities and between medium and large sized cities. There is a significant difference between group mean test scores for small and large sized cities, for small and extra large cities, for medium and extra large cities, and for large and extra large cities.

Per pupil expenditure

Ho: There is no significant difference between group mean test scores for Grade Four as measured by ELMR-4 when classified according to the expenditure per pupil for elementary school education in the school district.

Table 40. Summary of analysis of variance for per pupil expenditure for grade four

Source	SS	df	MS	F
Treatments	1551	2	775.7	
Within Treatments	39160	891	43.95	17.65**
Total	40710	893		

$F_{.01}(2, 891) = 4.64$ $F_{.05}(2, 891) = 3.01$

Examination of Table 40 and the critical F-ratio led to the rejection of the null hypothesis. Because of this, differences between pairs of means were analyzed using t-tests. Table 41 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-4 when classified according to expenditure per pupil for elementary school education in the school district.

- 1) H vs M
- 2) H vs L
- 3) M vs L

Table 41. Summary of tests for differences between pairs of group mean test scores for per pupil expenditure for grade four.

Group	N	Mean	Variance
H	208	26.39	49.60
M	393	23.50	40.95
L	293	25.95	43.51
Total	894	24.97	45.54

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_H - \bar{X}_M}{\sqrt{\frac{SS_H + SS_M}{n_H + n_M}}}} = 4.95^*$$

$$2) \quad t_{\frac{\bar{X}_H - \bar{X}_L}{\sqrt{\frac{SS_H + SS_L}{n_H + n_L}}}} = 0.72$$

$$3) \quad t_{\frac{\bar{X}_L - \bar{X}_M}{\sqrt{\frac{SS_L + SS_M}{n_L + n_M}}}} = 4.87^*$$

Based on the above, null hypotheses one and three were rejected while two was accepted. There is a significant difference between group mean test scores because of per pupil expenditure for elementary school education between those schools whose expenditure is high and those whose expenditure is medium and between schools with medium and low expenditures. There is no significant difference between schools whose expenditures are low and high.

Enrollment in elementary schools

Ho: There is no significant difference between group mean test scores for Grade Four as measured by ELMR-4 when classified according to the enrollment in the elementary schools of the district.

Table 42. Summary of analysis of variance for enrollment in elementary schools for grade four

Source	SS	df	MS	F
Treatments	44.56	2	22.28	
Within Treatments	40670	891	45.64	0.488
Total	40710	893		

$F_{.01}(2, 891) = 4.64$ $F_{.05}(2, 891) = 3.01$

Examination of Table 42 and the critical F-ratio indicates the acceptance of the null hypothesis. There is no significant difference between group mean scores because of the enrollment in the elementary schools of a district.

Per Pupil valuation of the school district

Ho: There is no significant difference between group mean test scores for Grade Four as measured by ELMR-4 when classified according to the valuation per pupil of the school district.

Table 43. Summary of the analysis of variance for the per pupil valuation for grade four

Source	SS	df	MS	F
Treatments	847.5	2	4238	
Within Treatments	39863	891	44.74	9.47**
Total	40710	893		

$F_{.01}(2, 891) = 4.64$ $F_{.05}(2, 891) = 3.01$

Examination of Table 43 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significance, differences between pairs of group mean test scores were analyzed using t-tests. Table 44 shows the information needed to complete the analysis of the following null hypothesis:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-4 when classified according to the valuation per pupil of the school district:

- 1) H vs M
- 2) H vs L
- 3) M vs L

Table 44. Summary of tests for differences between pairs of group mean test scores for per pupil valuation for grade four

Group	N	Mean	Variance
H	408	24.15	48.78
M	312	25.04	39.72
L	174	26.79	43.50
Total	894	24.97	45.54

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_M - \bar{X}_H}{\sqrt{\frac{48.78 + 39.72}{2}}}} = 1.98^*$$

$$2) \quad t_{\frac{\bar{X}_L - \bar{X}_H}{\sqrt{\frac{48.78 + 43.50}{2}}}} = 4.33^*$$

$$3) \quad t_{\frac{\bar{X}_L - \bar{X}_M}{\sqrt{\frac{39.72 + 43.50}{2}}}} = 2.61^*$$

Based on the above, are three of the null hypotheses were rejected. In the case of the first hypothesis, there is a very small significant difference between the means of the high and medium school districts. There is a significant difference between the means of schools with high and low valuations per pupil and between the means of schools with medium and low valuation.

Geographic location

Ho: There is no significant difference between group mean test scores for Grade Four as measured by ELMR-4 when classified according to the geographic location of the school district.

Table 45. Summary of analysis of variance for geographic location for grade four

Source	SS	df	MS	F
Treatments	860.2	3	430.1	
Within Treatments	39850	891	44.73	9.62**
Total	40710	893		

$F_{.01}(2, 891) = 4.64$ $F_{.05}(2, 891) = 3.01$

Examination of Table 45 and the critical F-ratio led to the rejection of the null hypothesis. Because of this, differences between pairs of group mean test scores were analyzed using t-tests. Table 46 shows the information needed to complete the analysis of the following null hypothesis:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-4 when classified according to the geographic location of the school district.

- 1) N vs C
- 2) N vs S
- 3) C vs S

Table 46. Summary of tests for differences between pairs of group mean test scores for geographic location for grade four

Group	N	Mean	Variance
N	178	26.34	45.15
C	278	25.62	35.28
S	438	24.01	50.24
Total	894	24.97	45.54

$$t_{.05^{\infty}} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\bar{X}_N - \bar{X}_S} = 1.17$$

$$2) \quad t_{\bar{X}_N - \bar{X}_S} = 3.85^*$$

$$3) \quad t_{\bar{X}_C - \bar{X}_S} = 3.29^*$$

Based on the above, the first null hypothesis was accepted while two and three were rejected. There is no significant difference between group mean test scores for schools in the northern and central sections of Wisconsin. However, there is a significant difference between schools in northern and southern Wisconsin and between schools in the central and southern parts of the state.

Sex

H₀: There is no significant difference between group mean test scores for Grade Four as measured by ELMR-4 when classified according to sex.

Table 47. Summary of analysis of variance for sex for grade four

Source	SS	df	MS	F
Treatments	188.8	1	188.8	
Within Treatments	40521	892	45.43	4.16*
Total	40710	893		

$$F_{.01}(1, 892) = 6.67$$

$$F_{.05}(1, 892) = 3.85$$

Examination of Table 47 and the critical F-ratio led to the rejection of the null hypothesis at the .05 level of significance. There is a significant difference between the group mean test score for boys and for girls. Table 48 shows the information needed to complete the analysis.

Table 48. Summary of tests for differences between pairs of group mean test scores for sex for grade four

Group	N	Mean	Variance
B	439	25.44	43.46
G	455	25.52	47.13
Total	894	24.97	45.54

Because of the significance of the difference between the group mean test scores, it can be seen that the mean score for girls is slightly higher than that for boys.

Grade five analysis of variance

The results of the analysis of variance for the six different null hypotheses for grade five are presented in this section.

Size of city or town

Ho: There is no significant difference between group mean test scores for Grade Five as measured by ELMR-5 when classified according to the size of the city or town on which the pupils attend school.

Table 49. Summary of the analysis of variance for size of city or town for grade five

Source	SS	df	MS	F
Treatments	3667	3	1222	
Within Treatments	51833	901	57.53	21.25**
Total	55500	904		

$F_{.01}(3, 901) = 3.81$ $F_{.05}(3, 901) = 2.61$

Examination of Table 49 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, difference between pairs of group mean test scores were analyzed using t-tests. Table 50 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-5 when classified according to the size of city or town in which the pupils attend school.

1) A vs B

2) A vs C

- 3) A vs D
- 4) B vs C
- 5) B vs D
- 6) C vs D

Table 50. Summary of tests for differences between pairs of group mean test scores for size of city or town for grade five

Group	N	Mean	Variance
A	166	27.11	51.83
B	106	24.53	70.36
C	451	23.70	61.05
D	182	20.66	45.26
Total	905	23.81	61.33

$$t_{.05^{\infty}} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_A - \bar{X}_B}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}} = 2.84^*$$

$$2) \quad t_{\frac{\bar{X}_A - \bar{X}_C}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_C^2}{n_C}}}} = 5.10^*$$

$$3) \quad t_{\frac{\bar{X}_A - \bar{X}_D}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_D^2}{n_D}}}} = 8.61^*$$

$$4) \quad t_{\frac{\bar{X}_B - \bar{X}_C}{\sqrt{\frac{S_B^2}{n_B} + \frac{S_C^2}{n_C}}}} = 0.93$$

$$5) \quad t_{\frac{\bar{X}_B - \bar{X}_D}{\sqrt{\frac{SS}{df}}}} = 4.05^*$$

$$6) \quad t_{\frac{\bar{X}_C - \bar{X}_D}{\sqrt{\frac{SS}{df}}}} = 4.90^*$$

Based on the above, all of the null hypotheses except four were rejected. Number four was accepted. Thus, in all cases, except the differences between the mean test scores for medium and large cities, the differences between the pairs of mean test scores was significant.

Per pupil expenditure

Ho: There is no significant difference between group mean test scores for Grade Five as measured by ELMR-5 when classified according to the expenditure per pupil for elementary school education in the school district.

Table 51. Summary of analysis of variance for per pupil expenditure for grade five

Source	SS	df	MS	F
Treatments	1782	2	890.5	
Within Treatments	53720	902	59.56	14.96**
Total	55502	904		

$F_{.01}(2, 902) = 4.63$ $F_{.05}(2, 902) = 3.01$

Examination of Table 51 and the critical F-ratio led to the rejection of the null hypothesis. Because of this, differences between pairs of means were analyzed using t-tests. Table 52 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of groups mean test scores as measured by ELMR-5 when classified according to expenditure per pupil for elementary school education in the districts:

- 1) H vs M
- 2) H vs L
- 3) M vs L

Table 52. Summary of tests for differences between pairs of group mean test scores for per pupil expenditures for grade five

Group	N	Mean	Variance
H	195	26.59	71.22
M	403	23.08	54.86
L	307	23.07	57.72
Total	905	23.81	61.33

$$t_{.05^{\infty}} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_H - \bar{X}_M}{\sqrt{\frac{S_H^2 + S_M^2}{2}}}} = 4.81^*$$

$$2) \quad t_{\frac{\bar{X}_H - \bar{X}_L}{\sqrt{\frac{S_H^2 + S_L^2}{2}}}} = 4.61^*$$

$$3) \quad t_{\frac{\bar{X}_M - \bar{X}_L}{\sqrt{\frac{S_M^2 + S_L^2}{2}}}} = 0.029$$

Based on the above, null hypotheses one and two were rejected, while three was accepted. There is a significant difference between

group mean test scores of school whose per pupil expenditure is high and those whose expenditure per pupil is medium and also between high and low schools. However there is no significant difference between group mean test scores for schools whose expenditures are medium and low.

Enrollment in elementary school

Ho: There is no significant difference between group mean test scores for Grade Five as measured by ELMR-5 when classified according to the enrollment in the elementary schools of the district.

Table 53. Summary of analysis of variance for enrollment in elementary schools for grade five

Source	SS	df	MS	F
Treatments	2883	2	1442	
Within Treatments	52620	902	58.34	24.71**
Total	55500	904		

$F_{.01}(2, 902) = 4.63$ $F_{.05}(2, 902) = 3.01$

Examination of Table 53 and the critical F-ratio led to the rejection of the null hypothesis. Because of this, differences between pairs of group mean test scores were analyzed using t-tests. Table 54 shows the information needed to complete the analysis of the following null hypotheses:

- 1) L vs M
- 2) L vs S
- 3) M vs S

Table 54. Summary of tests for differences between pairs of group mean test scores for enrollment in elementary schools for grade five

Group	N	Mean	Variance
L	655	22.83	57.05
M	106	24.53	70.36
S	144	27.72	54.13
Total	905	23.81	61.33

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in the chapter, the following were computed:

$$1) \quad t_{\bar{X}_M - \bar{X}_L} = 1.96$$

$$2) \quad t_{\bar{X}_S - \bar{X}_L} = 7.19^*$$

$$3) \quad t_{\bar{X}_S - \bar{X}_M} = 3.13^*$$

Based on the above, null hypothesis one was accepted while null hypotheses two and three were rejected. There is no significant difference between group mean test scores when the school enrollments one large and medium. There is a significant difference between mean test scores when the enrollments are large and small and when they are medium and small.

Per pupil valuation of the school district

Ho: There is no significant difference between group mean test scores for Grade Five as measured by ELMR-5 when classified according to the valuation per pupil of the school district.

Table 55. Summary of the analysis of variance for the per pupil valuation for grade five

Source	SS	df	MS	F
Treatments	268.6	2	134.3	
Within Treatments	55232	902	61.23	2.19
Total	55500	904		

$F_{.01}(2, 902) = 4.63$ $F_{.05}(2, 902) = 3.01$

Examination of Table 55 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference between group mean test scores when classified according to the per pupil valuation of the school district.

Geographic location

Ho: There is no significant difference between group mean test scores for Grade Five as measured by ELMR-5 when classified according to the geographic location of the school district.

Table 56. Summary of the analysis of variance for geographic location for grade five

Source	SS	df	MS	F
Treatments	352.8	2	176.4	
Within Treatments	55150	902	61.14	2.885
Total	55500	904		

$F_{.01}(2, 902) = 4.63$ $F_{.05}(2, 902) = 3.01$

Examination of Table 56 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference between the group mean test scores when classified according to the geographic location of the school district in the State of Wisconsin.

Sex

Ho: There is no significant difference between group mean test scores for Grade Five as measured by ELMR-5 when classified according to sex.

Table 57. Summary of analysis of variance for sex for grade five

Source	SS	df	MS	F
Treatments	0.6875	1	0.6875	
Within Treatments	55500	903	61.46	0.011
Total	55500	904		

$F_{.01}(1, 903) = 6.67$ $F_{.05}(1, 903) = 3.85$

Examination of Table 57 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference between group mean test scores when classified according to sex.

Grade six analysis of variance

The result of the analysis of variance for the six different null hypotheses for grade six are presented in this section.

Size of city or town

Ho: There is no significant difference between group mean test scores for Grade Six as measured by the ELMR-6 when classified according to the size of the city or town in which the pupils attend school.

Table 58. Summary of the analysis of variance for size of city or town for grade six

Source	SS	df	MS	F
Treatments	2013	3	670.9	
Within Treatments	42660	918	46.47	14.44**
Total	44670	921		

$F_{.01}(3, 918) = 3.81$ $F_{.05}(3, 918) = 2.61$

Examination of Table 58 and the critical F-ratio led to the rejection of the null hypothesis. Because of this significant relationship, differences between pairs of group mean test scores were analyzed using t-tests. Table 59 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-6 when classified according to the size of the city or town in which the pupils attend school:

- 1) A vs B
- 2) A vs C
- 3) A vs D
- 4) B vs C
- 5) B vs D
- 6) C vs D

Table 59. Summary of tests for differences between pairs of group mean test scores for size of city or town for grade six

Group	N	Mean	Variance
A	193	18.63	37.08
B	76	21.01	34.93
C	467	19.73	56.90
D	186	16.22	33.73
Total	922	18.89	48.45

$$t_{.05^{\infty}} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\bar{X}_B - \bar{X}_A} = 2.95^*$$

$$2) \quad t_{\bar{X}_C - \bar{X}_A} = 1.95$$

$$3) \quad t_{\bar{X}_A - \bar{X}_D} = 3.06^*$$

$$4) \quad t_{\bar{X}_B - \bar{X}_C} = 1.69$$

$$5) \quad t_{\bar{X}_B - \bar{X}_D} = 5.998^*$$

$$6) \quad t_{\bar{X}_C - \bar{X}_D} = 6.38^*$$

Based on the above, null hypotheses two and four were accepted, while one, three, five, and six were rejected. There is no significant

difference between group mean test scores between small and large sized cities nor between medium and large sized cities. There is a significant difference between group mean test scores for small and medium sized cities, between small and extra large cities, between medium and extra large cities, and between large and extra large cities.

Per pupil expenditures

Ho: There is no significant difference between group mean test scores for Grade Six as measured by ELMR-6 when classified according to the expenditure per pupil for elementary school education in the school district.

Table 60. Summary of analysis of variance for per pupil expenditure for grade six

Source	SS	df	MS	F
Treatments	1917	2	958.5	
Within Treatments	42753	919	46.52	20.60**
Total	44670	921		

$F_{.01}(2, 919) = 4.63$ $F_{.01}(2, 919) = 3.01$

Examination of Table 60 and the critical F-ratio led to the rejection of the null hypothesis. Because of this, differences between pairs of group mean test scores were analyzed using t-tests. Table 61 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-6 when classified according to the expenditure per pupil for elementary school education in the school district:

- 1) H vs M
- 2) H vs L
- 3) M vs L

Table 61. Summary of tests for differences between pairs of group mean test scores for per pupil expenditure for grade six

Group	N	Mean	Variance
H	206	21.28	64.74
M	420	17.58	37.72
L	296	19.10	45.86
Total	922	18.89	48.45

$$t_{.05^\infty} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_H - \bar{X}_M}{\sqrt{\frac{64.74 + 37.72}{2}}}} = 5.91^*$$

$$2) \quad t_{\frac{\bar{X}_H - \bar{X}_L}{\sqrt{\frac{64.74 + 45.86}{2}}}} = 3.18^*$$

$$3) \quad t_{\frac{\bar{X}_L - \bar{X}_M}{\sqrt{\frac{37.72 + 45.86}{2}}}} = 3.09^*$$

Based on the above, all of the null hypotheses were rejected. There is a significant difference between group mean test scores because of the expenditure per pupil between school districts.

Enrollment in elementary schools

Ho: There is no significant difference between group mean test scores for Grade Six as measured by ELMR-6 when classified according to the enrollment in the elementary schools of the district.

Table 62. Summary of analysis of variance for enrollment in elementary schools for grade six

Source	SS	df	MS	F
Treatments	371.9	2	186.0	
Within Treatments	44300	919	48.20	3.86*
Total	44670	921		

$F_{.01}(2, 919) = 4.63$ $F_{.05}(2, 919) = 3.01$

Examination of Table 62 and the critical F-ratio led to the rejection of the null hypothesis at the .05 level of significance. Because of this, difference between pairs of group mean test scores were analyzed using t-tests. Table 63 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-6 when classified according to the enrollment in the elementary schools for grade six:

- 1) L vs M
- 2) L vs S
- 3) M vs S

Table 63. Summary of tests for differences between pairs of group mean test scores for enrollment in elementary schools for grade six

Group	N	Mean	Variance
L	675	18.70	51.62
M	76	21.01	34.93
S	171	18.71	39.78
Total	922	18.89	48.45

$$t_{.05}^{\infty} = 1.96$$

Using the formula for calculating t discussed earlier in this chapter, the following were computed:

$$1) t_{\bar{X}_M - \bar{X}_L} = 3.16^*$$

$$2) t_{\bar{X}_S - \bar{X}_L} = 0.02$$

$$3) t_{\bar{X}_M - \bar{X}_S} = 2.76^*$$

Based on the above, null hypothesis two was accepted, while one and three were rejected. There is no significant difference between group mean test scores of elementary schools large enrollment and those with a small enrollment. However, there is a significant difference between the group mean test scores of a large and a medium enrollment and between those with a medium and a small enrollment.

Per pupil valuation of the school districts

Ho: There is no significant difference between group mean test scores for Grade Six as measured by ELMR-6 when classified according to the valuation per pupil of the school district.

Table 64. Summary of the analysis of variance for the per pupil valuation for grade six

Source	SS	df	MS	F
Treatments	371.1	2	185.6	
Within Treatments	44300	919	48.20	3.85*
Total	44670	921		

$F_{.01}(2, 919) = 4.63$ $F_{.05}(2, 919) = 3.01$

Examination of Table 64 and the critical F-ratio led to the rejection of the null hypothesis at the .05 level of significance. Because of this, difference between pairs of group mean test scores were analyzed using t-test. Table 65 shows the information needed to complete the analysis of the following null hypotheses:

Ho: There is no significant difference between the following pairs of group mean test scores as measured by ELMR-6 when classified according to the valuation per pupil of the school district:

- 1) H vs M
- 2) H vs L
- 3) M vs L

Table 65. Summary of tests for differences between pairs of group mean test scores for per pupil valuation for grade six

Group	N	Mean	Variance
H	449	18.50	54.11
M	320	19.76	46.02
L	153	18.25	34.50
Total	922	18.89	48.45

$$t_{.05^\infty} = 1.96$$

Using the formula for calculating t discussed earlier in the chapter, the following were computed:

$$1) \quad t_{\frac{\bar{X}_M - \bar{X}_H}{\sqrt{\frac{S^2}{N}}}} = 2.43^*$$

$$2) \quad t_{\frac{\bar{X}_H - \bar{X}_L}{\sqrt{\frac{S^2}{N}}}} = 0.43$$

$$3) \quad t_{\frac{\bar{X}_M - \bar{X}_L}{\sqrt{\frac{S^2}{N}}}} = 2.48^*$$

Based on the above, null hypothesis two was accepted while one and three were rejected. There is no significant difference between group mean test scores of districts whose valuation per pupil is high and those whose valuation is low. However, there is a significant difference between those districts whose valuation is high and those whose valuation is medium. Also, the difference between the means of districts whose valuation is medium and those whose valuation is low is significant.

Geographic location

Ho: There is no significant difference between group mean test scores for Grade Six as measured by ELMR-6 when classified according to the geographic location of the school district.

Table 66. Summary of analysis of variance for geographic location for grade six

Source	SS	df	MS	F
Treatments	77.44	2	38.72	
Within Treatments	44593	919	48.52	0.798
Total	44670	921		

$F_{.01}(2, 919) = 4.63$ $F_{.05}(2, 919) = 3.01$

Examination of Table 66 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference between group mean test scores for grade six when classified according to the geographic location of the school district in the State of Wisconsin.

Sex

Ho: There is no significant difference between group mean test scores for Grade Six as measured by ELMR-6 when classified according to sex.

Table 67. Summary of analysis of variance for sex for grade six

Source	SS	df	MS	F
Treatments	5.063	1	5.063	
Within Treatments	44670	920	48.55	0.104
Total	44670	921		

$F_{.01}(1, 920) = 6.67$ $F_{.05}(1, 920) = 3.85$

Examination of Table 67 and the critical F-ratio led to the acceptance of the null hypothesis. There is no significant difference between the group mean test scores for boys and girls at grade six.

DISCUSSION

The objectives of this study were 1) to determine whether or not the general notions identified by a factor analysis are the same as those which were indicated by the test committee and 2) to determine what, if any, differences between group mean test scores exist because of the size of the city or town in which the pupils attend school, the expenditure per pupil of the school district, enrollment in the elementary schools, per pupil valuation of the district, the geographic location of the district, or the sex of the pupils.

For grade one, the results of the factor analysis are shown in Table 68 with the items from the test shown under the general strands which the test committee identified in Table 69.

Table 68. Factors and test items for grade one

Factor	Test Items	Factor Name
I	59, 60, 61, 62	Basic subtraction
II	3, 34	Insert the missing factor
III	28, 29, 30, 31	Basic addition
IV	9, 15, 16, 17	Geometry
V	19, 20, 50, 51	Writing the number sentence
VI	14, 45	Relationship between addition and subtraction
VII	4, 35, 57	Renaming
VIII	5, 6, 10, 37	Place value

Table 69. Strands and test items for grade one

Strand	Strand Name	Test Items
1	Sets and numbers	1, 2, 3, 11, 32, 33, 34
2	Numeration systems	4, 5, 6, 9, 10, 35, 36 37, 40, 41, 42
3	Order	12, 44
4	Number systems	7, 8, 12, 14, 38, 39, 43, 45
5	Ratio	
6	Computation	28, 29, 30, 31, 59, 60 61, 62
7	Size and shapes	15, 16, 46, 47
8	Sets of points	17, 18, 48
9	Symmetry	
10	Congruence	
11	Similarity	
12	Coordinate systems	19, 20, 50, 51
13	Construction	
14	Measurement	21, 22, 23, 24, 49, 52 53, 54, 55
15	Mathematical Sentences	25, 26, 27, 28, 29, 30, 31, 56, 57, 58, 59, 60, 61, 62

(Some of the items may appear under two different strands in Table 69 because they may involve a mathematical sentence as well as a computation.)

By comparing Tables 68 and 69, it can be seen that all of the

items in factors one and three are in strands six and fifteen. This is true because basic fact computation as well as the idea of writing mathematical sentences are involved.

Both of the items in factor II are in the same general strand, one, of the test committee's breakdown. Numbers and their relation to sets of objects are involved in the objectives.

Four items loaded on factor III. They all were assigned to strand six by the test committee. Basic addition is tested by these items.

The four items loading on factor IV come from three different general strands, item nine from strand two, items fifteen and sixteen from strand seven, and item seventeen from strand eight. Because item nine asks the pupils to write a numeral while the other items require a knowledge of geometric notions, something other than these notions tended to bring these four items into the same factor. Difficulty or the time at which the topics were taught might have accounted for the combination of these items.

All four of the items in factor V are listed by the test committee as testing the same general strand. The students must identify the number sentence which tells the same story as the picture.

The two items which loaded on factor VI are included under the same general strand by the test committee. However, the committee indicates that six other items on the test are testing the same general concept. There are quite a variety of possible specific

objectives which might be included under the general strand of number systems, operations, and their properties: One of these is the relationship between addition and subtraction.

Two of the items which loaded on factor VII are in the same general strand while item fifty-seven is listed in another. However, all three of these items appear to be testing the same concept; different names for the same total quantity. The test committee identified a total of twelve items which were to test the concepts of numeration systems. Four of them make up factor VII, one is listed in factor IV, and five of them do not load on any factor.

Four items, all involving the notion of place value, load on factor VIII. All four of these are listed in the numeration systems strand by the test committee. Thus, there is a distinction between renaming and place value which places these in two different factors.

At the grade one level, only one item is placed in a factor different from that identified by the test committee. Only twenty-six of the sixty-two items on the test were placed in factors by the factor analysis procedure. There would appear to be little or no correlation between the thirty-six items not placed in a factor.

For grade two, the results of the factor analysis is shown in Table 70 with the items of the test shown under the general strand which the test committee identified in Table 71.

Table 70. Factors and test items for grade two

Factor	Factor Names	Test Items
I	Additive identity	7, 52, 57
II	Operations	10, 41, 54, 86, 59, 60
III	Numeral from words	1, 3, 38, 39
IV	Ordinal pattern	8, 40, 42, 64
V	Order relation	17, 44
VI	Unit fractional part	14, 66
VII	Mathematical factor	35, 48
VIII	Addition and Subtraction of whole numbers	4, 5, 26, 27, 28, 29, 30, 31, 45, 49, 50, 51, 53
IX	Place value	15, 18, 68
X	General number pattern	9, 13, 65

Table 71. Strands and test items for grade two

Strand	Strand Name	Test Items
1	Sets and numbers	14, 21, 64, 66, 68
2	Numeration systems	1, 2, 3, 4, 5, 15, 18, 38, 39, 40, 41, 67
3	Order	6, 8, 9, 16, 17, 42, 43, 44, 63
4	Number systems	7, 19, 32, 54, 57, 58, 65
5	Ratio	
6	Computation	10, 12, 26, 27, 28, 29, 30, 31, 48, 49, 50, 51, 52, 53, 59, 60
7	Size and shape	20
8	Sets of points	33, 35, 36, 69, 70, 72, 73
9	Symmetry	
10	Congruence	34, 74
11	Similarity	
12	Coordinate systems	37
13	Construction	
14	Measurement	22, 23, 24, 45, 46, 47, 71
15	Mathematical sentences	11, 13, 25, 26, 27, 28, 29, 30, 31, 48, 49, 50, 51, 52, 53, 55, 56, 61, 62

Even though the three items which loaded on factor one involved number operations and computation, there was a more specific notion which they appear to be testing, namely the additive identity factor. Both four and fifty-seven are listed under strand four while fifty-two is under strand six.

The six items loading on factor II appear in Table 71 under four different general strands; two, four, six, and fifteen. Nevertheless, there is the common element of operations or computation in general which each of these items shares with the other five. It should be pointed out that factor VIII includes those items which involve addition and subtraction of whole numbers. Factor II includes items requiring more than one operation.

Four items load on factor III. All four of them appear under general strand two of the test committee's listing. Factor III was called the numeral from words factor while strand two is called numeration systems. The specific behavioral objective under this strand which these items test is "the students should be able to read and write numerals to 999" (Appendix A). Again, the factor has indicated a specific concept from a broader general strand.

Four items loaded on factor IV. These four come from strands one, two, and three as indicated by the test committee. They all ask about an ordinal number pattern, given a part of that pattern.

The two items loading on factor V both are listed under strand three by the test committee, as were items eight and forty-two of

factor IV. The difference which places them in different factors concerns the order relations of less than or greater than. These are specifically asked in factor V but not in factor IV.

Unit fractional part is the name given to Factor VI. They both belong to strand one along with three other items including sixty-four which was placed in factor IV by the factor analysis. In the case of the items of factor VI, both ask what part of a geometric figure is shaded. In each case one out of several parts is shaded, that being the topmost part.

The two items loading on factor VII are listed under two different strands by the test committee and test two completely different concepts. This factor was called a mathematical factor.

Thirteen items loaded on factor VIII. With the exception of items four and five, which were listed by the test committee under strand two, and item 45 which was listed under strand fourteen, all of the items were listed under strand six, computation. However, all thirteen of the items involve the addition or subtraction of whole numbers. Thus, they were placed in this factor.

Each of the three items loaded on factor IX ask the pupils to write the numeral representing a quantity of counting sticks grouped to indicate hundreds, tens, and ones. For this reason, this factor was called the place value factor. Item sixty-eight was placed in strand one while items fifteen and eighteen were both placed in strand two.

Three items loaded on factor X, each having been listed under a different strand by the test committee. There is a general number pattern notion which is involved in all three of them.

For grade two, several items were grouped with items different from those with which they were grouped by the test committee. Of the seventy-four items on the test, forty-two of them were grouped in factors by the factor analysis. There would appear to be a fairly high correlation between those items placed in specific factors. Also, there appears to be little or no correlation between the thirty-two items not included in a factor either with each other or with items listed in the various factors.

The factor analysis for the third grade yielded ten factors. The factors and the items loading on each factor are shown in Table 72. The general strands and the items testing these strands as listed by the test committee are shown in Table 73.

Table 72. Factors and test items for grade three

Factors	Factor Name	Test Items
I	Place value	1, 4, 6, 8, 9
II	Computation	14, 19, 33
III	Number property	21, 29, 36
IV	Number relation	16, 22, 38
V	Naming	37, 39, 46, 47
VI	Matching	28, 32, 41
VII		10, 40
VIII	Difficulty	34, 43, 45
IX	Mathematical	7, 42
X	Identifying fractional parts	23, 32

Table 73. Strands and test items for grade three

Strands	Strand Name	Test Items
1	Sets and numbers	17, 23, 27, 32
2	Numeration systems	1, 2, 4, 8, 14, 15, 21, 35
3	Order	10, 16, 44
4	Number systems	3, 18, 22, 29, 36, 40
5	Ratio	7, 19
6	Computation	11, 30, 33, 45, 46
7	Size and shape	11, 30, 33, 45, 46
8	Sets of points	12, 42, 47, 48
9	Symmetry	43
10	Congruence	25, 26
11	Similarity	5
12	Coordinate Systems	13, 37, 41
13	Construction	
14	Measurement	20, 28, 34, 39
15	Mathematical sentences	6, 9, 24, 31, 38

Five items load on factor I. Of these, four ask about place value, while item six asks for one of the factors in a multiplicative situation. Because these items were all relatively easy for the third grade and they are all listed early in the test, item six was put with the other four. Items one, four, and eight were listed under

strand two by the test committee while items six and nine were included under strand fifteen. With the exception of item six, place value is the common concept.

Each of the three items loaded on factor II was listed under a different strand by the test committee. There is the common mathematical concept of computation which is involved in all three of them, although each of them does involve another concept.

Factor III was called the number property factor. Items twenty-nine and thirty-six were assigned to this strand while item twenty-one was placed under the numeration systems strand. Expanded notation was the property involved in that one item.

The three items which loaded on factor IV were placed in three different strands by the test committee. Two of them, sixteen and thirty-eight, asked for a greater or less than relationship while twenty-two asked for the relationship between the picture and the mathematical sentence indicating an operation. Because of the 'relationship' idea the three items were placed in the same factor.

The test committee listed each of the four items loading on factor V under different general strands. Item thirty-nine differs from the other three in that the word "name" does not appear. Item forty-six differs from the other three in that no geometric concepts are involved. However, other than difficulty, the common concept is that of naming.

Each of the three items loading on factor VI ask the student to

match a picture with a mathematical sentence. The test committee placed them under three different general strands.

There were two items which loaded on factor VII. Item forty showed a positive loading while item ten loaded negatively. One, ten, asks about order while the other asks about an operation.

The three items which loaded on factor VIII all came from different general strands and all ask about different mathematical concepts. The only common element seems to be the difficulty of the items.

Two items load on factor IX, one positively, item 7, and the other negatively, item 42. They were listed by the test committee under different strands and other than the fact that they test mathematics, do not appear to have anything in common.

Both of the items which loaded on factor X were listed under strand one by the test committee. The student was to identify a fractional part of a geometric figure.

In grade three, ten factors were identified in the factor analysis. Three of them grouped what appeared to be unrelated items, factors VII, VIII, IX. Only factor X grouped items from one strand. Each of the other six factors grouped items from more than one strand together. However, these factors could individually be named as indicating some mathematical concept.

Thirty of the forty-eight test items on ELMR-3 were loaded on one of the ten factors. This represents a higher proportion than for

either grades one or two. Again, those items not loading on a factor appear to correlate little or not at all with the items loading on a factor or with those items not loading on a factor.

The factor analysis for the fourth grade yielded seven factors. The factors and the items loading on each factor are shown in Table 74. The general strands and the test items for each strand as listed by the test committee are shown in Table 75.

Table 74. Factors and test items for grade four

Factor	Factor Name	Test Items
I	Number property	21, 23, 26, 34
II	Basic multiplication	2, 3, 4
III	Mathematical	33, 40
IV	Multiple operations	1, 5, 6, 9, 10, 11, 13, 14, 15, 18, 22
V	Factors of a number	27, 36, 39
VI	Place value	20, 25
VII	Geometric	7, 12, 16, 31

Table 75. Strands and test items for grade four

Strands	Strand Names	Test Items
1	Sets and numbers	27, 36
2	Numeration Systems	8, 17, 28
3	Order	13, 23
4	Number systems	1, 10, 46
5	Ratio	7, 26, 30, 42
6	Computation	2, 3, 4, 20, 24, 25, 29, 32, 43, 45
7	Size and shape	31, 40
8	Sets of points	19, 44, 48, 49, 51
9	Symmetry	47
10	Congruence	39
11	Similarity	41
12	Coordinate systems	12, 21, 50
13	Construction	35
14	Measurement	5, 14, 16, 22
15	Mathematical sentences	6, 9, 11, 15, 18, 33, 34, 37, 38

By comparing Table 74 and 75, it can be seen that the four items which load on factor I all come from four different general strands. However, each of them involves a property of the number system and, therefore, are put in the same factor by the factor analysis.

The three items loading on factor II all were assigned to the

same general strand by the test committee, computation. In fact, they all test the same specific objective under that general strand, recall of basic multiplication facts. The test committee has identified seven other test items which belong to that same general strand. However, each of these seven items involves a more complicated computational process than the simple recall of basic facts.

Two items load on factor III. These two were assigned to different strands by the test committee, one testing an arithmetic notion and the other testing a geometric notion. However, it can be seen from table four that items six, seven, and thirty-one load as highly on this factor as do items thirty-three and forty. But, these three items load higher on some other factor.

Factor IV was called the multiple operations factor. The eleven items which loaded on this factor were assigned to four different general strands by the test committee. Although items five, fourteen, and twenty-two were assigned to the measurement strand, the necessity to perform some mathematical operation as well as the measurement concept was evident.

Items twenty-seven and thirty-six loaded equally on factor V with item thirty-nine loading only slightly less. Taking into consideration the difficulty level of these items, this factor was identified as the 'factor of a number.' Items twenty-seven and thirty-six were assigned to the same general strand by the test committee with item thirty-nine being subsumed under a different general strand.

Both of the items which loaded on factor VI were listed by the test committee as testing strand six. Both of these items ask the pupils to indicate the digit in a particular position in the result of either addition or subtraction. These two items were listed under the same general strand as the items loading on factor II. The two sets of items tested different specific behavior objectives.

The four items which loaded on factor VII were assigned to four different general strands by the test committee. Item seven loaded positively while items twelve, sixteen, and thirty-one loaded negatively. The last three items test a geometric concept, while item seven tests the equivalence of fractions. The difficulty level played a role in putting these particular items together.

With ELMR-4, all of the general strands identified by the writers of Guidelines to Mathematics: K-6 were tested, if by one item only. However, there is a different grouping of test items as a result of the factor analysis than that indicated by the test committee. There was some common concept other than that suggested by the test committee which brought the items together.

Twenty-nine of the fifty-one items on ELMR-4 were placed in factor analysis. This indicated a high correlation between those items with lesser correlations between other items. There was little or no correlation between the items not placed in factors among the items or with items placed in factors.

The factor analysis for the fifth grade yielded rather peculiar results. A series of analyses were tried, beginning with a two factor

analysis and ending with a ten factor analysis. In each analysis no common concept could be identified except for factors XIII and IX of the ten factor analysis. No factor names or common mathematical concepts could be indicated for any of the other eight factors. The results of the ten factor analysis are shown in Table 76. The general strands and the test items are shown in Table 77.

Table 76. Factors and test items for grade five.

Factor	Factor Name	Test Item
I		8, 28, 45
II		4, 7, 16, 31
III		6, 54
IV		1, 9, 26
V		10, 34, 53
VI		49, 50
VII		36, 50
VIII	Line relationship	40, 41
IX	Division	20, 25
X		19, 42

Table 77. Strands and test items for grade five

Strand	Strand Name	Test Items
1	Sets and numbers	7, 14, 20, 25, 36, 52
2	Numeration systems	18, 46, 50, 51, 56
3	Order	8
4	Number systems	4, 10, 13, 19, 26, 30, 34
5	Ratio	35, 53, 54
6	Computation	1, 15, 27, 31, 47
7	Size and shape	16, 21, 39
8	Sets of points	23, 33, 40, 41, 45, 49
9	Symmetry	32
10	Congruence	2
11	Similarity	48
12	Coordinate systems	3, 22, 37, 38
13	Construction	6, 12
14	Measurement	9, 11, 17, 24, 28, 29, 55
15	Mathematical sentences	5, 42, 43, 44

By comparing tables 76 and 77, it can be seen that for factors I, II, III, IV, V, VI, VII, and X, the items which load on each of these was assigned by the test committee to different general strands. Although, there is a correlation between those items assigned to the same factor by the factor analysis, it is based on

something other than a common mathematical concept.

The items not loading on any factor correlate very little or not at all with the other items on the test.

For grade six, seven factors were identified. The factors and the items loading on those factors are shown in Table 78. The general strands and the test items for each strand as listed by the test committee are shown in Table 79.

Table 78. Factors and test items for grade six

Factor	Factor Name	Test Items
I	Geometric	25, 38, 43
II	Decimal equivalent	2, 5
III	Renaming	4, 6
IV	Mathematical	18, 20, 30, 31, 39, 45
V	Integer	7, 12, 29
VI	Rational number operations	8, 9, 23, 26, 27, 28
VII	Multiplication of fractions	16, 17

Table 79. Strands and test items for grade six

Strand	Strand name	Test Items
1	Sets and numbers	1
2	Numeration systems	2, 3, 4, 5, 6
3	Order	7
4	Number systems	8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19
5	Ratio	21, 22, 23, 24, 25
6	Computation	26, 27, 28, 29
7	Size and shape	30, 31
8	Sets of points	32, 33, 34
9	Symmetry	36
10	Congruence	35
11	Similarity	
12	Coordinate Systems	37
13	Construction	40, 41
14	Measurement	38, 39, 42, 43, 44, 45, 46, 47, 48
15	Mathematical sentence	49

Examination of Tables 78 and 79 reveals that twenty-four of the forty-nine items loaded on one of the seven factors. All three of the items in factor I have a geometric notion in common. The test committee concluded that item twenty-five tests the notion of ratio and proportion rather than the concept of similarity of two geometric figures.

The items which loaded on factors II and III were all listed under strand two by the test committee. They involve notions of numeration systems but do test two different concepts, factor II tests decimal equivalence while factor III tests the renaming of numbers.

The six items loading on factor IV were identified with three different strands by the test committee. There seems to be no common mathematical concept which would link them except in pairs. Items eighteen and twenty ask about properties of the number system. Items thirty and thirty-one ask for the names of geometrical sets of points. Items thirty-nine and forty both question measurement concepts. This factor was called a mathematical factor for lack of a common concept.

Although the three items which loaded on factor V were listed under three different general strands by the test committee, each of them involves the system of integers. Thus, this factor was called the integer factor.

The test committee indicated that the six items loading on factor V were to test three different concepts from arithmetic. However, they all involve computation of rational numbers.

Both items sixteen and seventeen involve the multiplication of fractions. The test committee assigned them to strand four because they involve operations in the number system. Factor analysis

picked them out for factor VII because they test the multiplication of fractions and also because they were placed together on the test.

One of the philosophical goals of all modern mathematical programs for the elementary schools is to develop an understanding of the theory of mathematics. This is to be accomplished along with the development of the ability to compute. The group of educators who wrote the Guidelines to Mathematics: K-6 took this into consideration when they established the behavioral objectives designed to accomplish this general goal. When an attempt is made to test the achievement of these objectives, it is difficult to determine whether a child understands the theory or is simply able to compute with a great deal of accuracy. Thus, the question of "just what a particular test item is testing" becomes of primary importance.

Quite often the test committee indicated that an item was to test a particular mathematical concept only to have the factor analysis indicate, by the grouping, that the item might be testing some other concept. Also, the factor analysis separated some of the items which the test committee had suggested were testing the same mathematical notion into two or more different factors, thereby indicating another grouping.

For some of the items, the test committee indicated that two strands might be tested. Factor analysis put some of these items

with completely different items, thus indicating that another concept might be playing a role.

The test committee indicated that certain items were to test a particular general strand. An example of this would be strand fourteen of grade six. Nine test items are listed. Four of them were placed in two different factors with the other five not being loaded on any one factor. Further examination of those items not loading on a in the factor analysis should be made.

At the first and second grades, the factors tended to correspond more closely with the general strands than they did at the fifth and sixth grade levels. Mathematical concepts developed in the early grades tend to be more specific and limited in number and scope. This could explain the divergence at the upper grades.

Very few geometric factors were shown although the test committee indicated a number of items which would test these concepts. Examination of tests indicates that these are often optional concepts or are the last ones to be included in a text. Many teachers lack sufficient confidence or knowledge of the geometry and decide to omit this if it is at all possible. This could be an explanation for the lack of geometric factors.

In this section of this chapter, the results of the analysis of variance for the six general hypotheses will be discussed.

First, does the size of the city or town in which the pupils attend school have any relationship to the scores on the ELMR tests?

When the cities were grouped according to the four categories, small, medium, large and extra-large, the analysis of variance resulted in a significant difference between group mean test scores for all six grades. Upon examination of the six pairs of group mean test scores for each of the six grades, the only generalization that can be made is that whenever the extra-large city was paired with any of the others, the result was significant. It can be seen from the respective table for each of the grades that the mean for this category was lower than the mean for any of the other categories. This accounted for three of the six possible pairs of group means. Of the other three pairs, two of the differences were significant for grades one, two, and five, while only one was significant for grades three, four, and six. There was no discernible general pattern of significance of the difference between group means.

One of the explanations for the mean of the extra-large city being lower than that of the others is that it is the only one with what might be called inner core schools, or ghetto schools. The pupils in such schools usually do not achieve at as high a level as do the pupils in other schools.

The number of pupils in each class for each of the categories was not known. It is possible that in the larger cities or towns, more pupils were assigned to each teacher. This would permit less contact with each individual pupil by a teacher and thus a lower level of achievement.

Does the amount of money spent on each elementary school child have any effect on the ELMR test scores? For grades one and four the comparison of those schools whose per pupil expenditure was high with those schools whose expenditure was low was not significant while the other two comparisons for each grade were significant. For grades three and five all of the comparisons were significant except for the ones involving schools with a medium per pupil expenditure and those with a low expenditure. This was not significant for either grade. For the second and the sixth grades, all of the comparisons were significant.

The group mean test score for the group classified as having a medium expenditure per pupil for elementary school education was lower than either of the other two categories for all of the grades except for grade three. The school district with the ghetto schools was included in this group. This could have accounted for the low mean. Although the group mean test score for the low expenditure group was less than that of either of the other groups for grade three, it was not significantly lower than the mean score for the medium group. Because of the group mean test score of the medium category being lower than that of either of the other categories for all except grade three, it cannot be said that there is a direct relationship between per pupil expenditure for elementary school education and scores on the ELMR tests.

Does the total enrollment in the elementary schools of a

district influence the ELMR test scores? Examination of the group mean test scores for this classification shows that the differences between the means was not significant for grade four. However, in the case of the other five grades, with the exception of the fifth grade, the group mean for the medium sized districts were highest, the small sized districts next, and that of the large sized districts was lowest. For the fifth grade, the mean for the small sized districts was next. With the exception of the first grade, the difference between the lowest two group mean test scores was not significant.

It might be concluded that if a school district has an elementary school enrollment of between one thousand five hundred and five thousand, the pupils will score higher than if the enrollment is either greater or less than these numbers. Such districts may be in a better position to secure competent teachers, their classes may be smaller, they may be better able to purchase teaching materials than the other districts. The learning environment in such districts may be more conducive to the successful learning of mathematics than that of larger or smaller districts.

Does the valuation per pupil in the district influence the ELMR test scores? Wealth in a district seems to have an adverse effect on the scores on the ELMR tests. Those districts which have a high valuation per pupil appear to have the lowest test scores. This is true for all grades in which the difference between the group mean

test scores was significant, except for grade six. For that grade, the lowest mean score was from the category with the lowest per pupil valuation.

With the exception of grade four, if the difference between group mean test scores was significant, the mean for the medium category was the highest. For grade four, the mean of the lowest valuation per pupil was the highest.

Are the schools with a high per pupil valuation less prudent in the spending of their money? This could be one of the possible explanations for the lowness of the group mean test score. Staffing, teaching materials, and class size might be factors to be considered as contributing to this difference.

Does the geographic location of the school district in the State of Wisconsin influence the score on the ELMR test? For grades three, five and six, it does not. For grades one and two, the differences between the group mean test scores were barely significant. No pattern is discernible for the three grades with significant differences between group mean test scores. Schools in the central section were highest in grade one, lowest in grade two, and the middle for grade four. The southern group of schools was in the middle for grades one and two but the lowest for grade four. The northern group of schools was lowest for grade one, but highest for grades two and four. No definite statement can be made about the influence of the geographic location of the school district on the score of the ELMR

test.

Is there a difference between the scores for boys and for girls on the ELMR tests? At the fourth grade level, this difference was significant at the .05 level of significance but not at the .01 level. The score for the girls was slightly higher than the score for the boys. In general, however, it can be said that there is no significant difference between the boys mean score and the girls mean score.

SUMMARY

The general objectives of this study were 1) to determine whether or not the general notions identified by a factor analysis are the same as those which are indicated by the Wisconsin Laboratory School Mathematics Committee for the ELMR series of tests and 2) to determine what, if any, differences exist between group mean test scores because of the size of the city or town in which the pupils attend school, the expenditure per pupil of the school district, enrollment in the elementary schools, per pupil valuation of the district, the geographic location of the district, or the sex of the pupils.

A test for each of the grades, one through six, was constructed by the Wisconsin Laboratory School Committee to test each of the specific behavioral objectives for each of the grades as listed in the Guidelines to Mathematics: K-6. The committee indicated which objective was to be tested by each of the specific items on the test. This study indicated that the items, in general, were not grouped by a factor analysis in the same way that the committee had originally grouped them. This difference became greater for the upper grades.

Classroom groups of students from school districts within the State of Wisconsin were used to gather the data for the study. These were chosen on the basis of Department of Public Instruction information about the districts. The tests were administered in late April or early May to all of the pupils in the classroom selected for the study. The data were then put on computer cards so that a factor

analysis and an analysis of variance could be completed.

A summary of the results of the analysis of variance is shown in Table 80. S indicates that the difference between group mean test scores was significant while an N indicates no significant difference. This indicates that the size of the city or town in which the school district is located and the expenditure per pupil for elementary education accounted for differences in group mean test scores. However, examination of the tables for these classifications for the individual grades does not yield the same relationship for each grade. The classification with the highest group mean test score is not the same for each grade. No definite pattern was shown.

Table 80. Summary of the results of analysis of variance for the six hypotheses for the six grades

Classification	Grade					
	I	II	III	IV	V	VI
City or town	S	S	S	S	S	S
Expenditure	S	S	S	S	S	S
Enrollment	S	S	S	N	S	S _(.05)
Valuation	S	S	N	S	N	S _(.05)
Location	S _(.05)	S _(.05)	N	S	N	N
Sex	N	N	N	S _(.05)	N	N

For some of the grades, the enrollment and the valuation classifications were not significant or were significant at the .05 level but not at the .01 level. Again, for those grades at which these two classifications were significant, no pattern could be identified.

Considering the geographic location of the school district and the sex of the pupils, these seem to have less effect on the test scores than do the other classifications. Location is significant at both the .05 and .01 levels for grade four only, and is significant at the .05 level for grades one and two. However, the same arrangement of the groups is not present for both grades.

Group mean test scores are significantly different for grade four and then only at the .05 level. Boys and girls do not score differently on the ELMR series of tests.

Implications

Evaluation of achievement is an essential part of education today. More and more, educators are turning to statements of observable behavioral objectives to determine the learning activities in which pupils are to engage. The goals of modern mathematics are somewhat different from those of the more traditional programs. This study indicates that, although test items and behavioral objectives are paired by the builders of the test, concepts which the student recognizes may influence his answer and thus create a grouping of the test items which is different from that originally intended. This study also indicated that the more traditional concepts of manipulation of numbers play a dominant role in evaluation. Few of the theoretical properties of our number or numeration systems were evident in the factors identified by factor analysis. Considerably

more work must be done to state the behavioral objectives in more restrictive terms. At the same time, test builders must reevaluate the tests to be certain that each item is testing what the builder believes that it is. Because of the possibility of more than one mathematical concept being incorporated in a test item, educators are not sure of the concepts understood by elementary school pupils.

Geometric objectives and items were not identified by the factor analysis as much as the items on the ELMR tests indicate that they should have been. Tests should be reconstructed which would better indicate the presence of geometric concepts in the curriculum of the elementary school.

Why did so many of the items on each of the ELMR tests not load on any factor in the factor analysis? This question should be pursued further. An examination of the correlation of each item with each of the other items might result in an answer to that question.

Since the results of the analysis of variance were somewhat inconclusive, it is suggested that a replication of the study be conducted. Perhaps such factors as teacher ability, the availability of audio-visual materials, size of classes, and the ability of the pupils could be more adequately controlled so as to indicate their effect on the ELMR test scores. Does attendance at a school which is in the inner-core or ghetto area of a large metropolitan area

influence the test score? Such a question was not answered by this study.

It would appear that there is a great need for the refinement of the behavioral objectives for elementary school mathematics and the identification of test items which will evaluate the degree of attainment of these objectives.

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APPENDIX A.

BEHAVIORAL OBJECTIVES FOR GRADES KINDERGARTEN

THROUGH SIX FROM

GUIDELINES TO MATHEMATICS: K-6

Key Mathematical Content Objectives and Related Student Behavioral Objectives

In the following outline, an attempt has been made to point out where key content objectives of mathematics might be introduced and developed in grades K-6. The corresponding behavioral objectives suggest a sequence of development of these mathematical ideas which provides for their reinforcement and continuity throughout the K-6 mathematics program. By means of a sequence of development, key ideas can be extended to conform with the maturity and background experience of the student.

The suggested content objectives have been organized under fifteen main topics: Sets and Numbers; Numeration Systems; Order; Number Systems, Operations and Their Properties; Ratio and Proportion; Computation; Size and Shape; Sets of Points; Symmetry; Congruence; Similarity; Coordinate Systems and Graphs; Constructions; Measurement; and Mathematical Sentences.

It is not intended that the placement of topics in this outline be considered as the only correct arrangement or that all of the topics necessarily be taught at every grade level as presented here. The ideas listed for each grade level should be regarded as a *suggested* guide for introducing various topics; the outline is not intended to be all-inclusive. The teacher will find it necessary to alter the order of topics to meet the needs of the children or the needs of a particular group in a given classroom.

Furthermore, no fixed amount of time or emphasis has been suggested for any objective in this outline. A disproportionate amount of space has been devoted to the geometric objectives due to their recent introduction in elementary mathematics courses. Many of the geometric objectives can be attained in

a relatively short time as compared with the time and emphasis necessary to attain those objectives of elementary school arithmetic which are central to an understanding and use of the rational number system.

This outline should prove useful to the school administration, the mathematics curriculum committee, and the teachers of a school district in one or more of the following ways:

- As an orientation to the key content and behavioral objectives of the elementary school mathematics curriculum.
- As a source of knowledge of the related objectives of elementary school mathematics, their development and reinforcement throughout the entire K-6 mathematics program.
- As a means of evaluating the content and behavioral objectives of the present mathematics program of the school on the basis of the needs of the school community.
- As a guide for the preparation of a mathematics curriculum guide, K-6, for use by the school district.
- As an aid in the evaluation and selection of textbooks.
- As a guide in determining the concepts that need to be highlighted in inservice activities for teachers.
- As an aid in choosing standardized tests.

Throughout this outline, an asterisk (*) appearing under any behavioral objective indicates that the understanding and skills listed for that objective in previous grades are to be expanded and developed.

Mathematical Concepts

Arithmetic Concepts

I Sets and Numbers

A set is a collection of things. Sets may have many properties, but a property peculiar to a collection or set is that of *number*. When an orderly arrangement of these numbers has been made, one has the tools needed for counting. Numbers can be used in the cardinal sense, that is, to indicate numerosness. For example, a triangle has *three* vertices. They can also be used in the ordinal sense, that is, to indicate the arrangement of numbers by order. For example,

a student in *fifth* grade is studying page *100* (the *hundredth* page) of his textbook. Sometimes a number is used in the nominal sense, that is, as a name. A social security number is used this way.

In this guide, the concept of number is extended to include positive rational numbers and then positive and negative integers. These number concepts will be more fully developed in the junior and senior high school mathematics programs.

Students should be able to:

II Numeration Systems

Names are given to the cardinal numbers. These names are numerals and are used to convey the idea of number. Numerals are symbolized forms used for communication. A given number has many different names which can be used depending on

the idea one wishes to express. Organized methods of arranging the numerals represent the numeration systems of the past and present. Study of different numeration systems will give the student insight and appreciation of the more efficient ones.

Students should be able to:

III Order

The concept of order is a generalization made as a result of experience in comparing unmatched sets of objects to determine which has more members. Order is expressed in sentences by means of the

symbols $>$ (is greater than) and $<$ (is less than). The concepts of equality and of inequality (numbers that are not equal) are considered here for convenience.

Students should be able to:

IV Number Systems, Operations and Their Properties

A number system consists of three parts: a set of numbers, two operations defined on the numbers in the set, and the properties (laws or rules) of the operations.

Two number systems, the whole numbers and the positive rational numbers, are studied in detail in the elementary school. A third system, the integers, is introduced in the elementary school when the physical situation demands that the number system be extended to include negative numbers. The integers and a fourth number system, the real numbers, are studied in more detail in junior and senior high school. An understanding of the integer and real number systems and their properties depends on careful and complete development of the whole number and (positive) rational number systems in the elementary school.

The operations on numbers studied in the elementary school are addition and multiplication, and their respective inverses, subtraction and division.

The important and useful properties of the operations on all number systems studied in arithmetic are given here. They are:

Addition

Commutative: $a + b = b + a$

Associative: $(a + b) + c = a + (b + c)$

Identity element: $a + 0 = a$

Inverse element: $a + (-a) = 0$

Multiplication

Commutative: $a \times b = b \times a$

Associative: $(a \times b) \times c = a \times (b \times c)$

Identity element: $a \times 1 = a$

Inverse element: $a \times 1/a = 1, a \neq 0$

Property of zero: $a \times 0 = 0$

Distributive: $a \times (b + c) = (a \times b) + (a \times c)$

Students should be able to:

Behavioral Objectives

Kindergarten

Sets and Numbers

1. Identify two equivalent sets by placing the members of the set in one-to-one correspondence.
2. Select the set of objects associated with a given number.
3. Count orally by matching numerals with sets having a given number of objects.
4. Identify, without counting, the number of sets with two, three, or four objects.
5. Use such terms as *more than*, *as many as*, *fewer than* when comparing sets of objects.

Numeration Systems

1. Identify the numerals 0 through 9.

Order

1. Determine whether two sets are equivalent (can be matched or placed in a one-to-one correspondence).
2. Compare two non-matching sets of fewer than 10 objects and decide which set has more members and which set has fewer members.
3. Determine that 3 is greater than 2 and that 2 is less than 3 by comparing appropriate sets of objects and do this for any two numbers less than 6.
4. Utilize the idea "one more than" in organizing sets in the natural order.

Number Systems, Operations and Their Properties

1. Rearrange sets of objects to demonstrate the joining and separating of sets, and thereby develop a readiness for addition and subtraction.
2. Tell a story about a problem represented by the above activities.
3. Use objects to represent the "action" or conditions of a problem.

Behavioral Objectives

First Grade

Sets and Numbers

1. Count the members of a set containing one hundred or fewer members.
2. Demonstrate one-half, one-fourth of a physical unit.
3. Use the ordinal numbers through fifth.
4. Use "0" as the symbol for the number of elements in the empty set.
5. Insert missing sets, such as a set of 3 elements between a set of 2 elements and a set of 4 elements.

Numeration Systems

1. Read and write any numeral from 0 through 100.
2. Give different numerals for a given number such as 6 + 2, 10 - 2, and 8 for eight.
3. Read number words through *ten*.
4. Interpret the place-value concept for writing whole numbers to one hundred; such as, 89 is the same as 8 tens, 9 ones.

Order

1. Compare two non-matching sets of less than 100 objects to decide which set has fewer (more) members.
2. Determine that 8 is greater than 5 and that 5 is less than 8 by comparing appropriate sets of objects and do this for any two numbers less than 10.

Number Systems, Operations and Their Properties

1. Identify the process of addition through experience with joining two sets of objects.
2. Identify the process of subtraction through experience with separating a subset from a set of objects.
3. Recognize examples of the commutative property for addition in the set of whole numbers.
4. Demonstrate with sets of objects the relationship between such sentences as $4 + 2 = 6$, $6 - 2 = 4$, and $6 - 4 = 2$.
5. Use the symbols +, -, and = to form sentences such as $3 + 6 = \square$.

Behavioral Objectives

Second Grade

Sets and Numbers

- Determine the cardinality of a set to one thousand when the objects are already grouped. For example, 4 bundles of 100 sticks, 9 bundles of 10 sticks, and 4 sticks are the same as 494 sticks.
- Use ordinal numbers through tenth.
- Identify $1/2$, $1/4$, $1/3$ of a whole by using physical objects.

Numeration Systems

- Read and write any numeral through 999.
- Write many symbols for the same number; for example, $6 + 3$, $5 + 4$, $17 - 8$, and 9 for nine.
- Count by 5's, 10's, and 100's.
- Use bundles of sticks to demonstrate place value through 999; for example, $234 = 200 + 30 + 4$.
- Write three-digit numerals in expanded notation; for example, $765 = 700 + 60 + 5$.
- Use physical objects to demonstrate regrouping; for example, 2 bundles of 10 sticks and 16 sticks have the same number of sticks as 3 bundles of 10 sticks and 6 sticks. Also 4 bundles of 10 sticks and 8 sticks have the same number of sticks as 3 bundles of 10 sticks and 18 sticks.

Order

- Use the terms *greater than* and *less than*, and *equals* in sentences.
- Use symbols $>$, $<$, and $=$ in mathematical sentences.
- Insert missing numbers, such as 3 between 2 and 4.
- Name successors and predecessors of each number through 99.
- Determine the order of numbers through 99.

Number Systems, Operations and Their Properties

- Recognize that there is no largest whole number.
- Use the associative property of addition in the set of whole numbers; for example, $(3 + 4) + 5 = 3 + (4 + 5)$.
- Recognize that subtraction is not commutative.
- Identify the process of multiplication through experience with joining several equivalent sets of objects.
- Discover from the addition table number patterns through the sum 18.
- Recognize zero as the identity element for addition in the set of whole numbers and its special role in subtraction.

Behavioral Objectives

Third Grade

Sets and Numbers

1. Determine the cardinality of a set to 10,000 through appropriate experiences.
2. Use ordinal numbers beyond tenth.
3. Identify $\frac{2}{3}$ and $\frac{3}{4}$ of a whole.
4. Show $\frac{2}{4} = \frac{1}{2}$, etc., by the use of physical objects or pictures.

Numeration Systems

1. Read and write any numeral to 10,000.
2. Write many symbols for the same number, such as $7 + 5$, 4×3 , $10 + 2$, and 12 for twelve.
3. Interpret place value to 10,000.
4. Write four-digit numerals in expanded notation; for example, $4567 = 4000 + 500 + 60 + 7$.
5. Recognize that numerals such as 57 can be expressed as $40 + 17$.
6. Read and write roman numerals through X (10).

Order

1. Determine betweenness, greater than, or less than for numbers through 999.
2. Recognize greater than or less than for the fractions $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$ with physical objects. (Note that *fractions* is being used here for *rational numbers*.)

Number Systems, Operations and Their Properties

1. Use the commutative and associative properties of multiplication in the set of whole numbers; for example, $4 \times 3 = 3 \times 4$ and $(4 \times 3) \times 2 = 4 \times (3 \times 2)$.
2. Recognize the role of 1 as the identity element for multiplication in the set of whole numbers.
3. Recognize the distributive property of multiplication over addition in the set of whole numbers; for example, $3 \times (2 + 4) = (3 \times 2) + (3 \times 4)$.
4. Discover number patterns from the addition and multiplication tables.
5. Demonstrate the concept of division by partitioning a set into several equivalent subsets; for example, separate a set of 12 objects into sets of 3 objects.
6. Demonstrate with sets of objects the relationship between such sentences as $4 \times 7 = 28$, $28 \div 4 = 7$, and $28 \div 7 = 4$.

Behavioral Objectives

Fourth Grade

Sets and Numbers

1. Determine the factors of a counting number (a whole number other than zero).
2. Determine common factors of two counting numbers.

Numeration Systems

1. Read and write numerals as needed.
2. Interpret place value for large numbers.
3. Use roman numerals through XXV.

Order*

Number Systems, Operations and Their Properties

1. Recognize the inverse relation between addition sentences and two subtractions sentences, such as $725 + 342 = 1067$ and $1067 - 725 = 342$ and $1067 - 342 = 725$.
2. Recognize the special role of 1 as a divisor.
3. Use parentheses to show order of operation; for example, $2 + 4 \times 3 = 2 + (4 \times 3) = 14$ and $(2 + 4) \times 3 = 6 \times 3 = 18$.

Behavioral Objectives

Fifth Grade

Sets and Numbers

1. Identify prime numbers such as 2, 3, 5, 7, 11, 13, 17,
2. Find the prime factors of numbers through 100.
3. Identify composite numbers.
4. Determine the least common multiple of two counting numbers.
5. Determine the greatest common factor of two counting numbers.
6. Construct sets of equivalent fractions through working with sets of objects. An example of such a set is $\{2/3, 4/6, 6/9, 8/12, \dots\}$.

Numeration Systems

1. Write many names for the same rational number.
2. Work with bases, such as 3, 4, 5, 6, and 7, to demonstrate an understanding of the base of a numeration system.
3. Use simple exponents such as $10^2 = 100$, $3^2 = 9$ and express 300 as 3×10^2 .
4. Read and write simple decimals.
5. Read and write roman numerals.

Order

1. Determine greater than, less than, and betweenness for rational numbers.

Number Systems, Operations and Their Properties

1. Recognize the set of positive rational numbers (fractions) as an extension of the set of whole numbers.
2. Recognize that zero is the identity element for addition in the set of positive rational numbers as well as in the set of whole numbers.
3. Recognize that subtraction is not always possible in the set of positive rational numbers and in the set of whole numbers.
4. Use the commutative and associative properties for addition in the set of positive rational numbers.
5. Recognize that there is no smallest positive rational number.
6. Add, subtract, and multiply simple rational numbers by use of physical objects, diagrams, etc.

Behavioral Objectives

Sixth Grade

Sets and Numbers

1. Use negative numbers in many different situations.

Numeration Systems

1. Represent rational numbers by decimals and fractions.
2. Express large numbers by using scientific notation, such as the distance from earth to the sun as 9.3×10^7 miles.
3. Use exponential notation in representing numbers; for example, $2345 = 2 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 5$.
4. Demonstrate an understanding of the relationship between decimals and common fractions.

Order

1. Determine greater than, less than, and betweenness for (positive, negative, and zero) integers.

Number Systems, Operations and Their Properties

1. Recognize that $1/1$ or 1 is an identity element for multiplication in the set of rational numbers.
2. Recognize the multiplicative inverse (reciprocal) for every positive rational number except zero and use it in the division of rational numbers. For example, $1/2 \div 3/4 = 1/2 \times 4/3$.
3. Recognize that the operation of division is the inverse of multiplication in the set of positive rational numbers. For example, the sentences $3/4 \times 2/3 = 1/2$, $1/2 \div 3/4 = 2/3$, and $1/2 \div 2/3 = 3/4$ have this relationship.
4. Recognize that there is no smallest or largest rational number between two positive integers.
5. Recognize that the integers (positive and negative whole numbers and zero) are an extension of the whole numbers.
6. Find the additive inverse (opposite) for each integer by using the number line.
7. Recognize that the rational numbers (positive and negative whole numbers, positive and negative fractions, and zero) are an extension of the integers.
8. Recognize that finding an integral power of a number involves repeated multiplication of the same number. For example, $(2/3)^3 = 2/3 \times 2/3 \times 2/3$.
9. Use the commutative and associative properties of multiplication for rational numbers.
10. Use the distributive property of multiplication with respect to addition of rational numbers.
11. Use the commutative property of addition for integers.
12. Recognize that the rational number system is dense; that is, between each two different rational numbers, there is a rational number.

Mathematical Concepts

Arithmetic Concepts

V Ratio and Proportion

A ratio is a pair of numbers used to compare quantities or to express a rate.

Symbols commonly used for a ratio are (a,b) , $a:b$, and a/b . Note that the symbol (a,b) is also used to denote the coordinates of a point in a plane and that the symbol a/b is generally used to name a

rational number, when a and b are integers and b is not zero.

A proportion is a statement that two ratios are equivalent (that two pairs of numbers express the same rate). A proportion is written in the form $a/b = c/d$, when $a \times d = c \times b$.

Students should be able to:

VI Computation

A certain amount of proficiency in the use of the various algorithms of arithmetic is necessary. The grade placement used for these computational skills in this guide is only an approximation. The expected

use and recall of certain addition, subtraction, multiplication, and division facts for the various grade levels are listed under the behavioral objectives.

Students should be able to:

Geometric Concepts

VII Size and Shape

The classification of and distinguishing characteristics of simple two and three dimensional figures are determined by their size and shape. Informal

examination of the size and shape of geometric figures develops an awareness of geometric patterns.

Students should be able to:

VIII Sets of Points

Space can be considered as a set of points. Intuitively, a point represents and is represented by a position or location in space. Lines and planes are

subsets of space. Each line is a set of points of space and each plane is a set of points of space. These subsets and many of their properties should be intuitively presented.

Students should be able to:

IX Symmetry

Many geometric figures and designs have a kind of balance called symmetry. If a figure can be folded so that corresponding parts coincide, then it

has a line of symmetry. A figure has a center of symmetry if for every point of the figure there is a second point such that a line segment joining them is bisected by this center.

Students should be able to:

X Congruence

Intuitively two geometric figures are congruent if they "fit" each other exactly, that is, if they have the same size and shape. More precisely, two sets of points are congruent if there is a one-to-one cor-

respondence between them which preserves distances; that is, if two points of one set are one inch apart, the corresponding points of a congruent set are also one inch apart.

Students should be able to:

Behavioral Objectives

Kindergarten

Ratio and Proportion

Computation

Size and Shape

1. Recognize squares, rectangles, circles, and triangles.

2. Use the terms *inside*, *outside*, and *on* as related to these figures.

Sets of Points

Symmetry

Congruence

Behavioral Objectives

First Grade

Ratio and Proportion

Computation

1. Use the addition facts through the sum of 10 and the corresponding subtraction facts.
-

Size and Shape

1. Observe distinguishing features of spheres, rectangular prisms (boxes), cylinders, and other objects.
 2. Use the terms *round*, *face*, *edge*, *corner*, and *surface*.
-

Sets of Points

1. Recognize physical representations of points, line segments, and portions of a plane (flat surfaces).
 2. Recognize that squares, rectangles, triangles, and circles are closed curves and tell whether a point is inside, outside, or on such a curve.
-

Symmetry

Congruence

Behavioral Objectives

Second Grade

Ratio and Proportion

Computation

1. Recall the addition facts through the sum of 18 and the corresponding subtraction facts.
2. Use the vertical algorithm in the addition of

three addends with one-place numerals; for example: 3

4

+ 5

—

3. Use the multiplication facts through the product 18.
-

Size and Shape*

Sets of Points

1. Recognize a point as a position and a line segment or curve as a set of points.
2. Recognize a straight line as a set of points with no beginning and no end.

3. Recognize a simple curve (in a plane) as one that does not cross itself.
 4. Recognize closed and open simple curves.
 5. Recognize the inside and outside of simple closed curves.
-

Symmetry

Congruence

1. Recognize congruent, plane figures as figures which fit on one another.

2. Recognize congruent segments as segments having the same length.

Behavioral Objectives

Third Grade

Ratio and Proportion

1. Interpret simple ratio situations, such as 2 apples for 15¢, written $\frac{2}{15}$ (apples) $\frac{15}{15}$ (cents).
2. Recognize that ratios such as $\frac{2}{5}$ (pencils) and $\frac{4}{10}$ (pencils) are equivalent ratios (represent the same rate).

Computation



1. Use the multiplication facts with products through 45 and the corresponding division facts.
2. Use the vertical algorithm in addition and subtraction with two- and three-place numerals when regrouping may be necessary.
3. Use the vertical algorithm to carry out multiplication with multipliers less than 10 when regrouping may be involved.
4. Multiply mentally by 10 and by 100.
5. Estimate the sum of two numbers. For example, $287 + 520$ is approximately $300 + 500$ or 800.

Size and Shape*

Sets of Points

1. Recognize rays and angles.
2. Recognize that there is only one line through two points and that two lines can intersect at only one point.
3. Recognize that many lines may pass through a point.

Symmetry

1. Recognize symmetry with respect to a line by folding paper containing symmetrical figures such as   along their vertical axes of symmetry.

Congruence

1. Recognize congruent angles.
2. Recognize that a rectangular sheet of paper can be divided into two or more congruent parts through folding.

Behavioral Objectives

Fourth Grade

Ratio and Proportion

1. Make up sets of equivalent ratios for given physical situations, such as $1/2$, $2/4$, $3/6$, $4/8$,
2. Determine if two ratios are equivalent by using the property of proportions commonly called *cross multiplication*. For example, $3/4 = 9/12$ because $3 \times 12 = 4 \times 9$, whereas $6/7 \neq 7/8$ because $6 \times 8 \neq 7 \times 7$.
3. Find the missing whole number in two equivalent ratios like $2/3 = \square/9$ or $5/\square = 25/70$.
4. Use equivalent ratios to convert units of measure, such as (gallons) $1 = \frac{3}{\square}$ (gallons)
(pints) $8 = \frac{\square}{\square}$ (pints)
to find how many pints there are in 3 gallons.

Computation

1. Recall the multiplication facts through 10×10 .
2. Do column addition with several four-place or five-place addends.
3. Subtract using three-place numerals and four-place numerals.
4. Multiply a number by multiples of 10, by multiples of 100.
5. Use the multiplication algorithm with two-place multipliers.
6. Use the subtractive division algorithm with two-place divisors ending in 1, 2, 3, 4.
7. Estimate the product of two numbers and the quotient of two numbers. For example, 21×88 is approximately 20×80 or 1800 and $795 \div 23$ is approximately $800 \div 20$ or 40.

Size and Shape

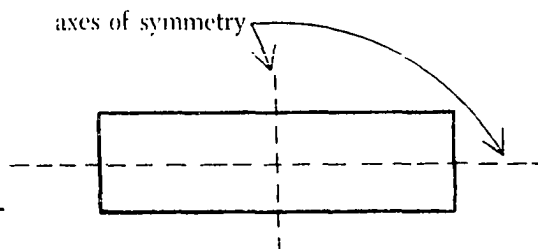
1. Recognize isosceles and equilateral triangles and parallelograms.

Sets of Points

1. Interpret space as the set of all points.
2. Recognize a plane as a flat surface which contains lines and points.
3. Describe lines as intersections of planes.
4. Interpret a circle as the set of all points in a plane that are at the same distance from a fixed point.
5. Recognize parallel lines as lines in a plane which do not intersect.

Symmetry

1. Recognize that some figures have two or more axes of symmetry through paper folding. For example, two axes of symmetry are indicated for the rectangle shown at right.



Congruence

1. Recognize congruent angles.

Behavioral Objectives

Fifth Grade

Ratio and Proportion

1. Use the ideas of ratio and equivalent ratio with problems that include fractions as terms. For example, find the missing number in $\frac{2/3}{5} = \frac{\square}{20}$.
2. Find the missing term in a proportion such as $2/5 = \square/9$ by using the cross multiplication property and find the solution of the mathematical sentence $18 = 5 \times \square$
3. Use members of sets of equivalent ratios with the same first term or the same second term to compare different ratios. For example, to compare $5/9$ and $3/4$, show $20/36 < 27/36$, and to compare $5/11$ and $3/7$, show $15/33 > 15/35$. Also, $5/9 < 3/4$ because $5 \times 4 < 9 \times 3$. Likewise $5/11 > 3/7$ because $5 \times 7 > 11 \times 3$.

Computation

1. Add and subtract rational numbers.
 2. Use the multiplication algorithm with numerals up to four places.
 3. Use the subtractive division algorithm.
 4. Express the quotient of integers as a mixed numeral; for example, $24 \div 5 = 4 \frac{4}{5}$.
 5. Find many ways to express a rational number; for example, $2/3 = \frac{2 \times 4}{3 \times 4} = 8/12$ and
- $$8/12 = \frac{8 \div 4}{12 \div 4} = 2/3.$$

Size and Shape

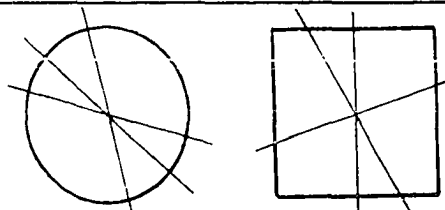
1. Recognize common polyhedra, such as a tetrahedron, a cube, a rectangular prism.
2. Identify faces, edges, vertices, and diagonals of common polyhedra.
3. Use Euler's formula, namely, $V + F = E + 2$ where V is the number of vertices, F is the number of faces, and E is the number of edges of any polyhedron.

Sets of Points

1. Recognize acute, right, and obtuse angles.
2. Recognize parallel planes.
3. Recognize perpendicular lines.
4. Recognize the radius and diameter of a circle.
5. Recognize that a plane is determined by three points not all on one line.

Symmetry

1. Recognize symmetry with respect to a point by folding a paper along a line through the center of such geometric figures as a circle and a square.



Congruence

1. Recognize that triangles are congruent if corresponding sides are congruent and corresponding angles are congruent.

Behavioral Objectives

Sixth Grade

Ratio and Proportion

1. Interpret percent as a ratio in which the second number is always 100.
2. Solve all three cases of percentage problems as problems in which they find the missing term of two equivalent ratios. For example, 20% of 30 and: $20/100 = \square/30$; 30 is what percent of 55 and: $\square/100 = 30/55$; 25 is 40% of what number and: $40/100 = 25/\square$.
3. Use equivalent ratios to convert fractions to decimals and conversely; for example, to write $3/5$ as hundredths, solve for n in $3/5 = n/100$; to write 44 hundredths as 25ths, solve for n in $n/25 = 44/100$.
4. Solve ratio problems where some or all of the terms of the ratios are written as decimals.
5. Use proportions in problems about the lengths of sides of similar triangles.

Computation

1. Multiply and divide rational numbers.
2. Use the conventional division algorithm.
3. Add integers.

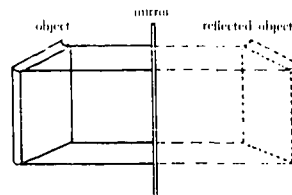
Size and Shape*

Sets of Points

1. Recognize the properties of isosceles triangles, equilateral triangles, and scalene triangles, such as the fact that the longest side of a triangle is opposite the angle of greatest measure.
2. Recognize that a line (one-dimensional space) is a subset of a plane (two-dimensional space) and that both are subsets of space (three-dimensional space).
3. Recognize the relationship between the circumference and the diameter of a circle.

Symmetry

1. Recognize the reflection of a plane figure in a mirror and draw diagrams such as the figure at right.



Congruence*

Mathematical Concepts

Geometric Concepts

XI Similarity

Two geometric figures that have the same shape, though not necessarily the same size, are said to be similar. The measures of corresponding dimensions of these figures will, thus, have the same ratio. Ideas

of similarity are needed for the interpretation and drawing of building plans, road maps, and scale models.

Students should be able to:

XII Coordinate Systems and Graphs

A line is called a number line if a one-to-one correspondence exists between a given set of numbers and a subset of points on the line and if, by means of this correspondence, the points are kept in the same order as their corresponding numbers. The

number corresponding to a point is the coordinate of that point. The idea of assigning numbers to points can be extended to points in a plane, that is, a one-to-one correspondence between ordered pairs of numbers and points in a plane.

Students should be able to:

XIII Construction

Students can gain insight to geometry by using a variety of materials and instruments to construct

models of geometric figures. Some useful materials and instruments are paper (for folding), wire, string, coins, rulers, compasses, and protractors.

Students should be able to:

XIV Measurement

The process of measuring associates a number with a property of an object. Measuring an object is done either directly or indirectly. In direct measurement the number assigned to an object is determined by its direct comparison to a selected unit of measure of the same nature as the object being measured. When a measuring instrument cannot be applied directly to the object to be measured, indirect measurement is employed.

Measure of physical objects is approximate. The accuracy of the measure obtained is restricted by the unevenness of the object measured, by the limitations of the measuring instrument used, and by human inabilities. The object to be measured must be measured by any unit with the same characteristics: a unit segment to measure segments; a unit angle to measure angles; a unit closed region to measure closed regions; and a unit solid to measure solids.

Students should be able to:

Mathematical Sentences

XV Mathematical Sentences

Mathematical sentences and ordinary linguistic sentences have the following common characteristics:

1. Both types of sentences use words (symbols) to communicate ideas.
2. The construction of both sentences follows a predetermined set of rules.
3. Both sentences are true statements or false statements depending on the words or symbols

used and the contexts in which they are used. Mathematical sentences like $2 + 3 = 5$ (an equation) and $3 + 4 < 6$ (an inequality) are either true or false. Thus the first sentence is true whereas the second one is false. Mathematical sentences like $\square + 3 = 7$ and $12 - N > 5$ are true for some replacements of \square and N and are false for others. If the set of positive integers is used as replacements for \square and N in the above sentences, then the respective solution sets are $\{4\}$ and $\{1, 2, 3, 4, 5, 6\}$.

Behavioral Objectives**Kindergarten****Similarity**

Coordinate Systems and Graphs

Construction

Measurement

1. Recognize pennies, nickels, dimes.
2. Make comparisons in time and count whole units

- of time (day, week, month, year).
3. Use appropriately such words as *longer, shorter, heavier, lighter, higher, lower, larger, smaller.*
-

Mathematical Sentences

Behavioral Objectives

First Grade

Similarity

Coordinate Systems and Graphs

1. Use the number line to illustrate addition and subtraction problems.

Construction

Measurement

1. Determine which of two line segments is the longer or the shorter, or whether they are the same length.
2. Recognize the comparative value of coins (pennies, nickels, dimes) and use them in making change.
3. Tell time to the nearest half-hour.
4. Identify various instruments of measurement of time, temperature, weight, and length, such as clocks, thermometers, scales, rulers.
5. Use non-standard units of linear measure and liquid measure, such as a pencil or book for length, and a paper cup for liquid measure.

Mathematical Sentences

1. Write an appropriate mathematical sentence like $3 + 4 = \square$ or $5 - 2 = \square$ for a physical situation or a story problem where the "action" of the problem suggests the operation of addition or subtraction.
2. Find the "solution" for sentences like $3 + 4 = \square$ and $5 - 2 = \square$.
3. Find solutions for sentences like $\square + \triangle = 7$ in which many correct solutions are possible.
4. Make up a problem situation to fit a given mathematical sentence involving addition or subtraction.

Behavioral Objectives

Second Grade

Similarity

Coordinate Systems and Graphs

1. Use the number line to illustrate counting by fives and by tens.

Construction

Measurement

1. Make a ruler with divisions showing half units.
2. Use standard units to the nearest whole unit for linear measure (inches and feet), for weight (pounds), and for liquid measure (pints and quarts).
3. Identify proper instruments for measuring different objects.
4. Tell time to the nearest quarter hour.
5. Make change correctly for quantities up to 25¢.

Mathematical Sentences

1. Use sentences like $5 + \square = 12$, $\square + 6 = 8$, $12 - \square = 8$, and $\square - 5 = 6$ to represent physical situations and find solutions for the sentences.
2. Make up story problems to fit sentences like those shown above.
3. Find solutions for sentences like $3 + 2 = 8 - \square$, $\square + 5 = 8 + 7$, $8 + \square < 12$, and $4 + 9 > \square + 5$, with the aid of sets of objects or the number line.
4. Place the correct symbol (+ or -) in the placeholder in sentences like $13 \triangle 5 = 8$ and $4 \square 3 < 5$.
5. Use sentences like $27 = 20 + 7$ and $43 = 30 + 13$ to indicate regrouping or the use of different symbols for the same number.
6. Use equivalent sentences like $3 + \square = 7$ and $7 - \square = 3$ to show subtraction as the inverse of addition.

Behavioral Objectives

Third Grade

Similarity

1. Recognize that figures are similar if they have the same shape. For example, all squares are similar.

Coordinate Systems and Graphs

1. Recognize that a point on a line can be described by a number (coordinate).
2. Recognize that different (uniform) scales can be

applied to the same line.

3. Use the number line to illustrate multiplication problems.

Construction

Measurement

1. Use common standard units such as inches, feet, yards, in determining the measure of a distance.
2. Use standard units of measure, such as cups,

gallons, ounces, in determining capacity and weight.

3. Find the perimeter of a rectangle or parallelogram.
4. Make change for any purchase up to \$1.00.

Mathematical Sentences

1. Use sentences like $3 \times 5 = \square$, $\square \times 7 = 14$, and $4 \times \square = 12$ to represent physical situations and find solutions for the sentences.
2. Make up story problems to fit sentences like those shown above.
3. Place the correct symbol ($<$, $>$, $=$) in the place holder in sentences such as $3 \times 5 \square 7 + 8$, $25 + 42 \square 87 - 28$, and $65 - 39 \square 5 \times 7$.
4. Demonstrate understanding of grouping and re-

grouping by completing sentences such as $458 = \square + 50 + 8$ and $394 = 3 \text{ hundreds} + \square \text{ tens} + 4 \text{ ones}$, by means of tally boxes or other devices.

5. Find solutions for sentences like $\square + 239 = 239 + \square$ and $1987 + (\square + 548) = (1987 + \square) + 548$ to generalize the idea of the commutative and associative properties for addition.
6. Use many different kinds of placeholders like \square , \triangle , N, X in mathematical sentences.

Behavioral Objectives

Fourth Grade

Similarity

1. Recognize that all congruent figures are similar, but not all similar figures are congruent.

Coordinate Systems and Graphs

1. Recognize that a line segment is a set of points.
2. Recognize that points in a plane (the first quadrant) can be represented by (ordered) pairs of numbers (coordinates).
3. Use the number line to illustrate division problems.

Construction

1. Demonstrate through paper folding an understanding of a line as an intersection of two planes.
2. Reproduce a line segment by using a compass and straightedge.
3. Bisect a line segment by using a compass and straightedge.

Measurement

1. Compare measures such as: 25 inches and 2 feet; 31 ounces and 2 pounds; 75 seconds and 1 minute; and 15 pints and 2 gallons.
2. Express different names for the same measure.
3. Measure perimeters of triangles and quadrilaterals.
4. Find areas of simple regions informally. For example, a rectangular region with dimensions 2" by 3" can be covered by six one-inch squares (regions).

Mathematical Sentences

1. Use sentences like $36 \div 4 = \square$ and $\square \times 3 = 12$ to represent physical situations and find solutions for the sentences.
2. Make up story problems to fit sentences like those shown above.
3. Use sentences like $\square \times 5 = 45$ and $45 \div 5 = \square$ to show division as the inverse of multiplication.
4. Find solutions for sentences like $723 \times \square = \square \times 723$ and $(\square \times 176) \times 19 = \square \times (176 \times 19)$ to generalize the idea of the commutative and associative properties of multiplication.
5. Find solutions for sentences like $3 \times 13 = (\square \times 10) + (\square \times 3)$, $\square \times \triangle = (5 \times 7) + (5 \times 8)$, and $\square \times 16 = (\square \times 10) + (\square \times 6)$ to generalize the distributive property of multiplication over addition.
6. Find solutions for sentences like $\square \times \triangle = 36$.
7. Recognize that $3 \times \square = 7$ has no whole number solution.
8. Find solutions for mathematical sentences involving more than one operation such as $(2 \times 5) + 4 = \square$ and $(3 \times 2) + \square = 10$.
9. Make up problem situations to fit mathematical sentences involving more than one operation. For example, make up a story to fit the sentence $(3 \times 4) + 2 = \square$.

Behavioral Objectives

Fifth Grade

Similarity

1. Recognize the similarity of maps made with different scales.

Coordinate Systems and Graphs

1. Construct simple picture, bar, and line graphs.
2. Use the number line to represent positive ra-

tional integers.

3. Use the number line to represent negative integers.

Construction

1. Demonstrate an understanding of various polyhedra by making appropriate paper models.
2. Bisect an angle. (Students may discover several

different constructions.)

3. Reconstruct an angle and a triangle by using a compass and a straightedge.

Measurement

1. Measure an angle by using a protractor.
2. Demonstrate that the sum of the measures of the angles of a triangle is 180° by tearing off and matching corners of a triangular piece of paper.
3. Find the area of a plane region, such as a rectangular region.

4. Recognize informal concepts of volume; for example, a box with dimensions 2" by 3" by 4" contains 24 one inch cubes.

5. Recognize that a right angle has the measure 90° .

6. Estimate distances to the nearest unit.

7. Recognize that all measurement involves approximation.

Mathematical Sentences

1. Use all of the previously introduced sentence forms with fractions.
2. Write sentences using fractions to represent physical situations.

3. Use previously described sentence forms to generalize the commutative property of addition, the associative property of addition, and the distributive property of multiplication over addition for rational numbers in fractional form.

Behavioral Objectives

Sixth Grade

Similarity*

Coordinate Systems and Graphs*

Construction

1. Construct a line perpendicular to a given line.
2. Construct parallel lines.
3. Make models of various prisms and find their surface areas.

Measurement

1. Find the volume of a rectangular prism.
2. Estimate and compare perimeters of polygons, such as rectangles, triangles, and parallelograms.
3. Estimate the area of an irregular plane region by use of a grid where an approximation to the area is the average of the inner and outer areas.
4. Use formulas for the areas of rectangles, parallelograms, and triangles.
5. Use the formula for the circumference of a circle.
6. Use the metric system of measure for length.
7. Use formulas of volume for common solids.
8. Work with approximate numbers. For example, know that the area of a square whose sides measure 6.5 and 3.6 inches to the nearest tenth of an inch has an area between 6.4×3.5 and 6.6×3.7 square inches.
9. Solve problems involving the measurement of inaccessible heights and distances indirectly by using the properties of similar triangles.

Mathematical Sentences

1. Use all of the previously introduced sentence forms with decimal numerals.
2. Write sentences using decimal numerals to represent physical situations.
3. Use previously described sentence forms to generalize the commutative property of multiplication, the associative property of multiplication, and the distributive property of multiplication over addition for rational numbers in any form.

APPENDIX B.

MEMBERS OF THE TEST COMMITTEE

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River Falls

Lynn Oberlin, Wisconsin State University--
Platteville

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Education District 8, Waukesha

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River Falls

Karl Zahn, Wisconsin State University--
Whitewater

APPENDIX C.

ELMR-1

WISCONSIN MATHEMATICS TEST

GRADES 1 - 6

(Based on the specific student behavioral objectives in Guidelines to Mathematics, K-6, Bulletin Number 141)

Issued by

William C. Kahl, State Superintendent
Wisconsin Department of Public Instruction

Bulletin No. 1626

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FOREWORD

Contemporary mathematical content objectives, agreed upon by a group of respected Wisconsin mathematics educators, were identified in Guidelines to Mathematics, K-6, published three years ago by the Department. We are pleased now to publish a mathematics achievement test for grades 1-6 that is based on those objectives. School districts that accept for their programs the objectives listed in our guide are encouraged to reproduce and use this test.

This achievement test can be used only to assess the accomplishment of cognitive objectives. Since we feel that the affective impact is of equal importance, it should be noted that appreciation of mathematics, liking to do mathematics, and a desire to explore mathematics, are desirable outcomes of a well-balanced school mathematics program.

Our appreciation is extended to the members of the Wisconsin Laboratory School Mathematics Committee for the many long hours of hard work put in during the development of the Wisconsin Mathematics Test.

William C. Kahl

State Superintendent

Directions for Administering Test

(To be administered in two sittings separated by a recess period.)

MATERIALS

Test booklet
Soft lead pencil
Eraser
Marker (Made from tagboard 2 inches wide and 9 inches long.)

FRONT PAGE

I am going to give each of you a booklet with some arithmetic problems for you to do. Do not open your booklet until I tell you to do so.

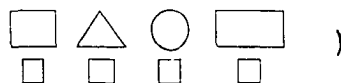
PASS OUT THE TEST BOOKLETS AND MARKERS

Look at the front page. Find the star and the word Name. Put your marker on the line under the star. Write your first name and last name on the line after the word Name.

SAMPLE A

We are going to do some arithmetic problems together. Move your marker down to the line under row A.

(Have Sample A on the chalkboard



Put a cross in the box under the circle.

(Show how to mark with the sample on the chalkboard.)

(Check to see that each child follows directions for marking.)

SAMPLE B

Move your marker down to the line under row B.

(Check to see that the markers are in the correct place.)

Look at the numerals in row B. Put a cross in the box under the numeral 8. (Repeat) The numeral 8.

SAMPLE C

Look across the page at the next problem, problem C.

Look at the number word. Put a cross in the box beside the numeral that is the same as the number word. (Repeat directions)

(Check to see that each child follows directions.)

NOW TURN THE PAGE

I will read the directions. You will put a cross in the box under the right answer. Listen carefully to what I say. You may not be able to answer all the questions but try to do your best work.

PART ONE (First Sitting)Page 1Behavioral Objectives

PUT YOUR MARKER ON THE LINE UNDER THE FIRST PROBLEM.

(Check to see that each child has the correct item.)

1. Look at the set of blocks. Count the blocks in the set. 1 - 1 - 1
Put a cross in the box under the numeral that tells the number of blocks.

MOVE YOUR MARKER DOWN TO THE LINE UNDER THE NEXT PROBLEM.

2. Look at the sticks. Put a cross in the box under the stick that is one-half shaded. 1 - 1 - 2
(Repeat) One-half shaded.

MOVE YOUR MARKER DOWN TO THE LINE AT THE BOTTOM OF THE PAGE.

3. Look at the sets of apples in the top row. One of the sets is missing. Put a cross $1 - 1 - 5$ in the box under the set of apples that should be put in the place of the missing set. (Repeat)

TURN THE PAGE

Page 2

PUT YOUR MARKER ON THE LINE UNDER THE TOP ROW OF PROBLEMS.

4. Look at the first problem. Beside the boxes are names for numbers. Put a cross in $1 - 2 - 2$ the box beside the name for 7. (Repeat) The name for 7.
5. Look across the page at the next problem. Look at the numerals. Put a cross in $1 - 2 - 1$ the box under the numeral 16. (Repeat) The numeral 16.

MOVE YOUR MARKER DOWN TO THE LINE UNDER THE NEXT PROBLEMS.

6. Put a cross in the box under the numeral that shows 3 tens and 7 ones. (Repeat) $1 - 2 - 4$
3 tens and 7 ones.
7. Look across the page at the next $1 - 4 - 3$ problem. Look at the 2 plus 5. 2 plus 5 is a name for a number. Which choice is another name for the same number? Put a cross in the box beside that choice.

MOVE YOUR MARKER DOWN TO THE LINE AT THE BOTTOM OF THE PAGE.

Page 2 (continued)

8. The picture shows that 2 chicks joining 3 chicks will make a set of 5 chicks. Put a cross in the box beside the word that tells the kind of a sentence you would write for this picture. 1 - 4 - 1

Subtraction. (Pause) Addition.
(Repeat the directions)

TURN THE PAGE

Page 3

PUT YOUR MARKER ON THE LINE UNDER
THE TOP ROW OF PROBLEMS.

9. In the box with the ball, write the numeral 60. (Repeat) 60. 1 - 2 - 1
10. Look across the page at the next problem.
Look at the 57. Put a cross in the box under the numeral that is in the tens place. (Repeat the directions.) 1 - 2 - 4

MOVE YOUR MARKER DOWN TO THE
LINE UNDER THE NEXT PROBLEMS.

11. Look at the set of stars. The numeral 5 tells the number of members in the set of stars. Now look at the other set. Write the numeral in the box that tells the number of members in that set. 1 - 1 - 4
12. Look across the page at the next problem. Look at the number sentence 5 circle 2 equals 3. The circle shows a sign is missing. Put a cross in the box beside the missing sign. 1 - 4 - 5

Page 3 (continued)

MOVE YOUR MARKER DOWN TO THE LINE UNDER
THE NEXT PROBLEM.

13. Look at the sets of dots. Count the dots in each set. Put a cross in the box under the set that has more dots. 1 - 3 - 1

MOVE YOUR MARKER DOWN TO THE
LINE AT THE BOTTOM OF THE PAGE.

14. Look at the set of rabbits and the number sentence under it. On the line write a subtraction sentence that goes with the addition sentence 4 plus 2 equals 6. (Repeat the directions: "On the line...")
(The answer can be either $6-2=4$ or $6-4=2$.) 1 - 4 - 4

TURN THE PAGE

Page 4

PUT YOUR MARKER ON THE LINE UNDER
THE TOP ROW OF PICTURES.

15. Look at the pictures of the objects. Put a cross under the object that has no edges. (Repeat) No edges. 1 - 7 - 1
and
1 - 7 - 2

MOVE YOUR MARKER DOWN.

16. Look at the pictures of the objects. Put a mark in the box under the cone. (Repeat) The cone. 1 - 7 - 1

MOVE YOUR MARKER DOWN.

17. Look at the pictures. Put a cross in the box under the line segment. (Repeat) The line segment. 1 - 8 - 1

MOVE YOUR MARKER DOWN.

Page 4 (continued)Behavioral
Objectives

18. Look at the triangles. Put a cross in the box under the triangle that has a point on the triangle. (Repeat the directions) 1 - 8 - 2

TURN THE PAGE

Page 5

PUT YOUR MARKER ON THE LINE UNDER
THE PROBLEM

19. Look at the number line and the arrows below it. Put a cross in the box beside the number sentence that this number line shows. 1 - 12 - 1

MOVE YOUR MARKER DOWN.

20. Look at the number line and the arrows below it. Put a cross in the box beside the number sentence that this number line shows. 1 - 12 - 1

MOVE YOUR MARKER DOWN.

21. Look at the line segments. Put a cross in the box beside the choice that is true. I will read the choices. (Read the choices to the children.) Listen carefully. 1 - 14 - 1

AB is the longer segment.
CD is the longer segment.
AB is the same length as CD.

TURN THE PAGE

Page 6

PUT YOUR MARKER ON THE LINE SO YOU CAN SEE
ALL THE ROWS OF PENNIES.

Page 6 (continued)

22. Look at the nickel. Put a cross in the box beside the row of pennies that is the same value as one nickel. (Repeat the directions.) 1 - 14 - 2

MOVE YOUR MARKER DOWN.

23. Look at the clocks. Put a cross in the box under the clock that shows three thirty or half past three. 1 - 14 - 3

MOVE YOUR MARKER DOWN.

24. Look at the pictures. Put a cross under the picture of the object that can tell how much a bag of apples weighs. 1 - 14 - 4

TURN THE PAGE

Page 7

PUT YOUR MARKER ON THE LINE UNDER THE PROBLEMS.

25. I am going to read a story problem. Listen carefully. 1 - 15 - 1

Bill had 2 marbles. His friend gave him 5 marbles. How many marbles does Bill have now?

Put a cross in the box beside the number sentence that tells the story of the problem. (Repeat the story problem from "Bill had...")

26. Look across the page at the next problem. Look at the number sentences. Which sentence is not true? Put a cross in the box beside the number sentence that is not correct. (Repeat)
- 1 - 15 - 3

MOVE YOUR MARKER DOWN TO THE
LINE UNDER THE PICTURES.

27. Look at the four pictures of rabbits. Put a cross in the box under the picture that tells the story of the number sentence 3 minus 1 equals 2. (Repeat)
- 1 - 15 - 4

MOVE YOUR MARKER DOWN TO THE
BOTTOM OF THE PAGE.

28. Thru 31. Look at the problems next to the ball. You will add in all the problems. Write the answers in the boxes. Begin working.
- 1 - 6 - 1
and
1 - 15 - 2
- (Allow time to finish.)

THE FIRST TESTING PERIOD ENDS HERE.
COLLECT TEST BOOKLETS AND MARKERS.

PART TWO (Second Sitting)

We are going to do more arithmetic problems. Open your booklet to page 8. I will read the directions. You will put a cross in the box under the right answer. Listen carefully.

Page 8Behavioral Objectives

PUT YOUR MARKER ON THE LINE UNDER THE ROW OF RABBITS. (Check to see that each child has the correct item.)

32. Look at the gate and the rabbits. Put a cross in the box under the third rabbit from the gate. 1 - 1 - 3

MOVE YOUR MARKER DOWN TO THE LINE UNDER THE NEXT ROW.

33. Look at the squares. Put a cross in the box under the square that is one-fourth shaded. (Repeat) One-fourth shaded. 1 - 1 - 2

MOVE YOUR MARKER DOWN TO THE LINE AT THE BOTTOM OF THE PAGE.

34. Look at the sets of flowers in the big box. The set that should come between these two sets is missing. Find the missing set in the row below. Put a cross in the box under the missing set. The set that belongs between the two sets. 1 - 1 - 5

TURN THE PAGE

Page 9

PUT YOUR MARKER ON THE LINE UNDER THE PROBLEMS.

35. Look at the first problem. Beside the boxes are names for numbers. Put a cross in the box beside a name for eight. (Repeat) A name for 8. 1 - 2 - 2

36. Look across the page at the next problem. Put a cross in the box beside the word five. (Repeat) Five. 1 - 2 - 3

MOVE YOUR MARKER DOWN.

37. Look at the numerals. Put a cross in the box under the numeral that shows 1 ten and 4 ones. (Repeat) 1 - 2 - 4

38. Look across the page at the next problem. Look at the number sentence 3 plus 5 circle 8. The circle shows that a sign is missing. Put a cross in the box beside the sign that will make a true sentence. (Repeat) 1 - 4 - 5

MOVE YOUR MARKER DOWN TO
THE LINE AT THE BOTTOM OF
THE PAGE

39. The picture shows that separating 1 balloon from 4 balloons leaves a set of 3 balloons. Put a cross in the box beside the word that tells the kind of sentence you would write for this picture. Subtraction. (Pause) Addition. (Repeat the directions) 1 - 4 - 2

TURN THE PAGE

PUT YOUR MARKER ON THE LINE UNDER
THE BOXES.

40. In the box with the tree, write
the numeral 19. (Repeat) 1 - 2 - 1
19.

41. Look across the page at the
next problem. In the box with
the lollipop, write the numeral
74. (Repeat) 74. 1 - 2 - 1

MOVE YOUR MARKER DOWN.

42. Look at the 18. Put a cross in
the box under the numeral that
is in the ones place. 1 - 2 - 4
(Repeat directions.)

43. Look across the page at the
next problem. Look at the 1 - 4 - 5
number sentence 4 circle 2
equals 6. The circle shows
a sign is missing. Put a cross
in the box beside the missing
sign.

MOVE YOUR MARKER DOWN.

44. Look at the sets. Count the
members in each set. (Pause
for counting.) Put a cross 1 - 3 - 2
in the box in front of the
sentence that is true. I will
read the sentences. 7 is
less than 5. 7 is equal to 5.
7 is greater than 5. (Repeat
the answers.)

MOVE YOUR MARKER DOWN TO THE
BOTTOM OF THE PAGE.

45. Look at the set of rabbits and the
number sentence under it. On
the line write an addition sentence
that goes with the 1 - 4 - 4

subtraction sentence 5
minus 2 equals 3. (Repeat
the directions from "On the
line...") (The answer
can be either $2+3 =5$ or
 $3+2=5$)

TURN THE PAGE

Page 11

PUT YOUR MARKER ON THE LINE
UNDER THE TOP ROW OF PICTURES

46. Look at the pictures of the objects.
Put a mark in the box under the 1 - 7 - 1
object that has a cross on one and
edge. (Repeat) One edge. 1 - 7 - 2

MOVE YOUR MARKER DOWN.

47. Look at the pictures of the ob-
jects. Put a mark in the box 1 - 7 - 1
under the cube. (Repeat) The
cube.

MOVE YOUR MARKER DOWN.

48. Look at the figures. Put a
cross in the box under the 1 - 8 - 2
closed curve. (Repeat)
The closed curve.

MOVE YOUR MARKER DOWN.

49. Look at the stick and the
pencil. About how many pen-
cils can be laid end-to-end
along the stick? Put a cross 1 - 14 - 5
in the box under the numeral.
(Repeat directions.)

TURN THE PAGE

PUT YOUR MARKER ON THE LINE UNDER THE PROBLEM.

50. Look at the number line and the arrows below it. Put a cross in the box beside the number sentence that this number line shows. 1 - 12 - 1

MOVE YOUR MARKER DOWN.

51. Look at the number line and the arrows below it. Put a cross in the box beside the number sentence that this number line shows. 1 - 12 - 1

MOVE YOUR MARKER DOWN.

52. Look at the line segments. Put a cross in the box beside the choice that is true. I will read the choices. (Read the choices to the children.) Listen carefully. 1 - 14 - 1

EF is the shorter segment.
GH is the shorter segment.
EF is the same length as GH.

TURN THE PAGE

Page 13

PUT YOUR MARKER ON THE LINE UNDER ALL THE ROWS OF MONEY

53. I am going to read a story problem. Listen carefully. 1 - 14 - 2

Tom bought a toy car for 7 cents. He gave the storekeeper a dime. How much change did he get?

Put a cross in the box beside the change Tom got back.

(Repeat the story problem from "Tom bought...")

MOVE YOUR MARKER DOWN.

54. Look at the clocks. Put a cross in the box under the clock that shows six o'clock. (Repeat) Six o'clock. 1 - 14 - 3

MOVE YOUR MARKER DOWN.

55. Look at the pictures. Put a cross under the picture of the object that can show the temperature. 1 - 14 - 4

TURN THE PAGE

Page 14

PUT YOUR MARKER ON THE LINE UNDER THE PROBLEMS.

56. I am going to read a story problem. Listen carefully. 1 - 15 - 1

Jim had 4 pieces of candy.
He ate 2 pieces of candy.
How many pieces of candy does he have now?

Put a cross in the box beside the number sentence that tells the story of the problem. (Repeat the story problem from "Jim had...")

57. Look across the page at the next problem. Look at the number sentences. Which sentence is not true? Put a cross in the box 1 - 15 - 3

beside the number sentence that is
not correct. (Repeat)

MOVE YOUR MARKER DOWN TO THE
LINE UNDER THE PICTURES.

58. Look at the four pictures of birds.
Put a cross in the box under 1 - 15 - 4
the picture that tells the story
of the number sentence 2 plus
3 equals 5. (Repeat)

MOVE YOUR MARKER DOWN TO THE
BOTTOM OF THE PAGE.

59. Thru 62. Look at the problems
next to the balloon. You will 1 - 6 - 1
subtract in all the problems. and
Write the answers in the boxes. 1 - 15 - 2
Begin working. (Allow time to
finish.)

END OF SECOND TESTING PERIOD.
COLLECT TEST BOOKLETS AND MARKERS.

MATHEMATICS TEST
FIRST GRADE

Name _____ last _____ first _____

Boy _____ Girl _____ Grade _____

School _____ Teacher _____

City _____ State _____

Date of Testing _____
year month day

Date of Birth _____
year month day

Age _____
years months

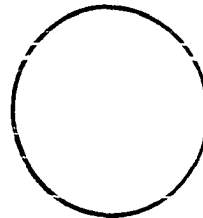
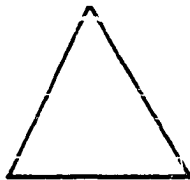
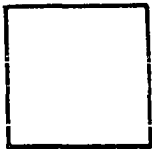
TEST SCORES	
Number correct	_____
Number wrong	_____
Number omitted	_____
Total test items - 62	



Name _____

Samples

A



B

6



7



8



9



C

two



0



1



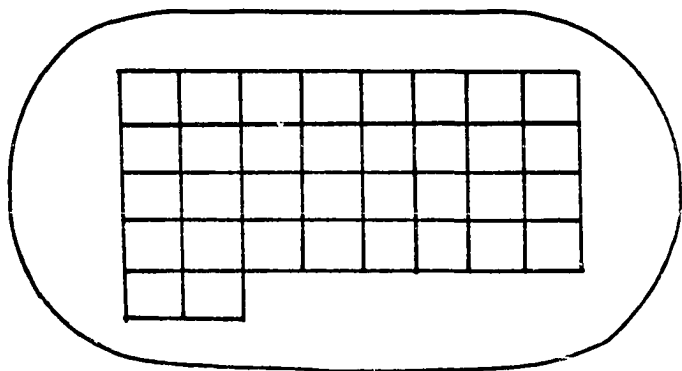
2



3

Part One ²³⁵

1



33

34

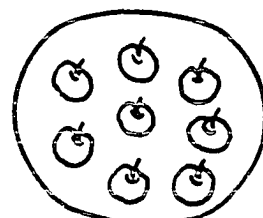
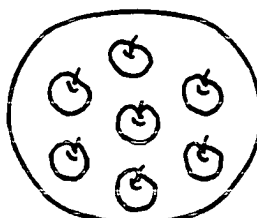
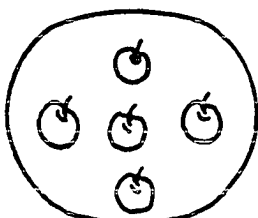
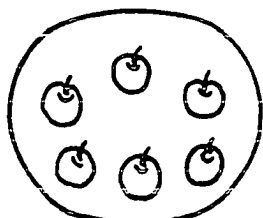
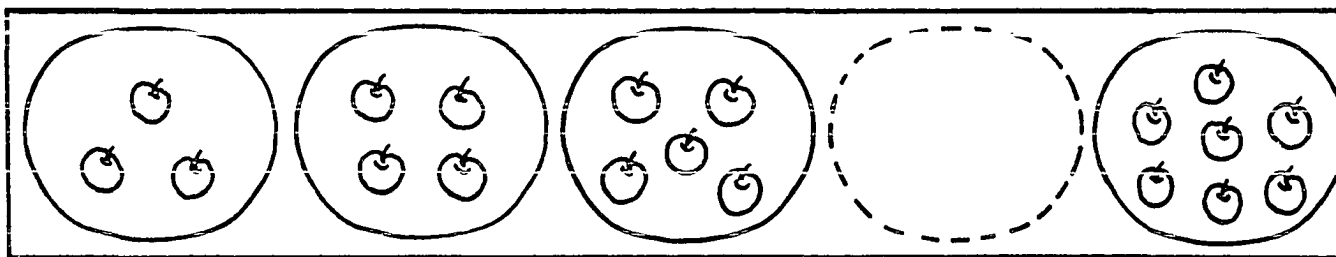
35

45

2



3



4

$$\square 3 + 4$$

$$\square 8 - 2$$

$$\square 1 + 7$$

$$\square 7 - 2$$

5

19

16

66

61

6

17

23

37

73

7

$$\square 2 + 3$$

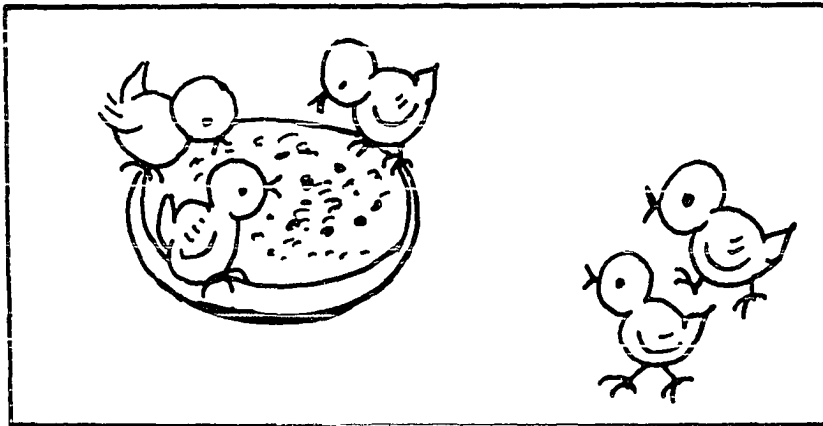
$$\square 5 + 3$$

$$2 + 5$$

$$\square 2 + 4$$

$$\square 5 + 2$$

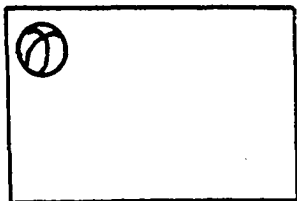
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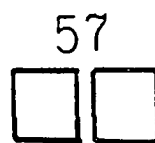
Subtraction

Addition

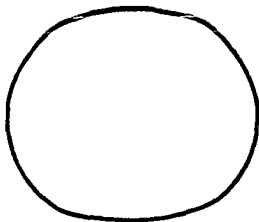
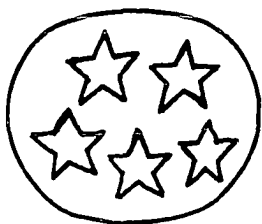
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10



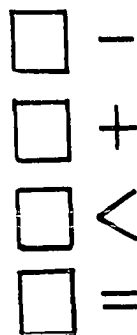
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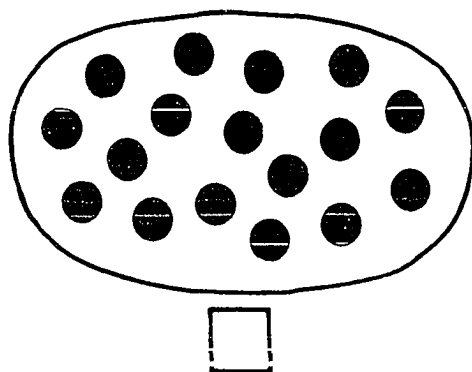
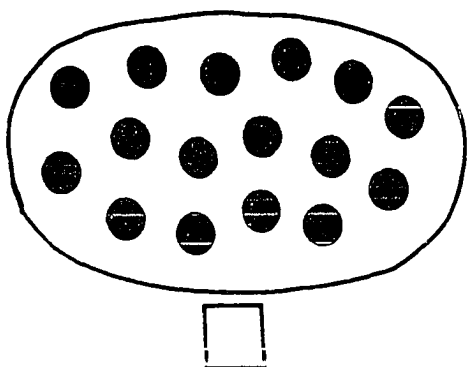


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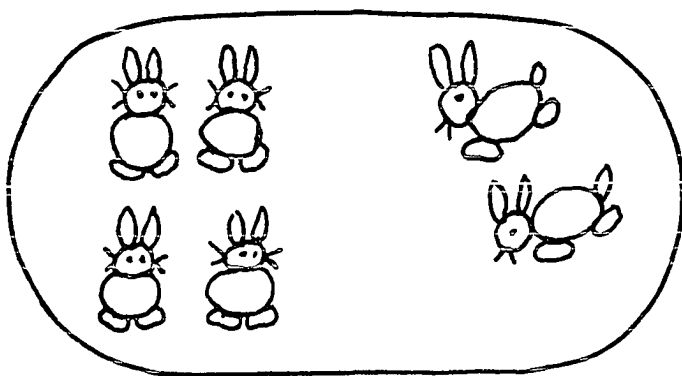
$$5 \bigcirc 2 = 3$$



13



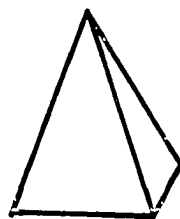
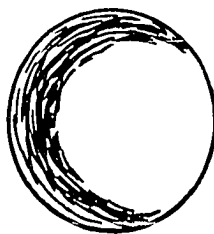
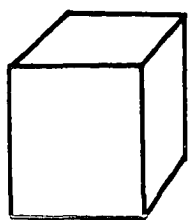
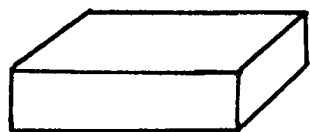
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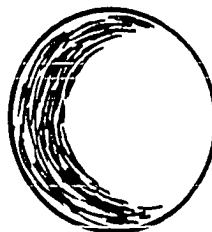
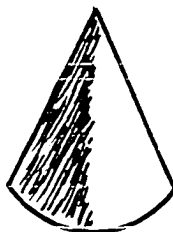
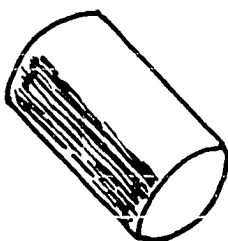
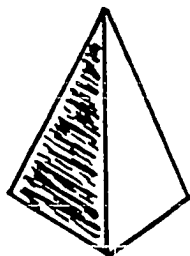
$$4 + 2 = 6$$



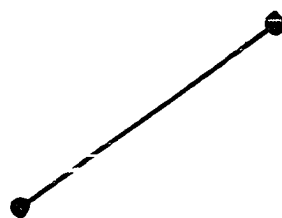
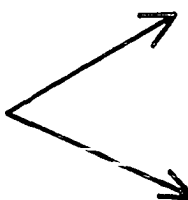
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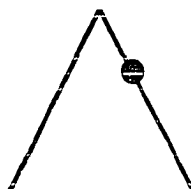
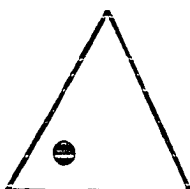
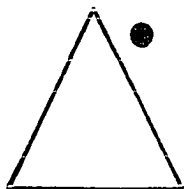
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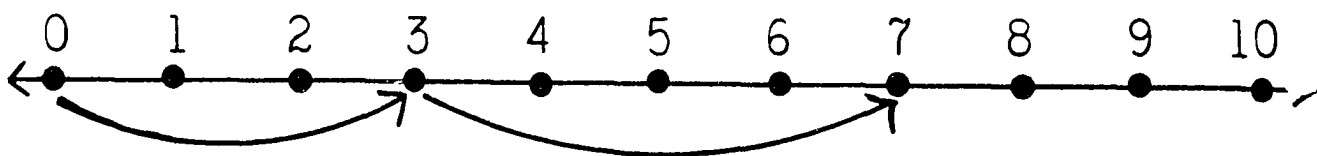
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18



19



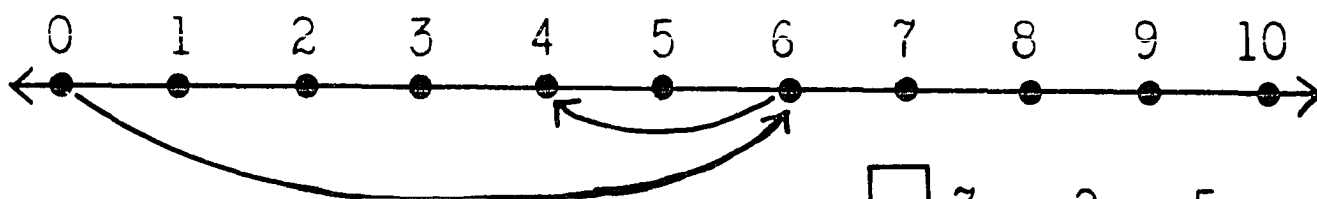
$3 + 5 = 8$

$5 + 2 = 7$

$3 + 4 = 7$

$4 + 2 = 6$

20



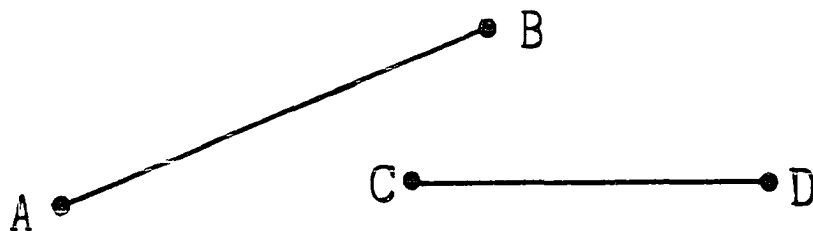
$7 - 2 = 5$

$5 - 1 = 4$

$5 - 3 = 2$

$6 - 2 = 4$

21


 AB is the longer segment.

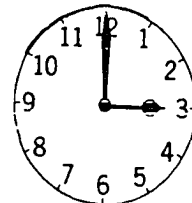
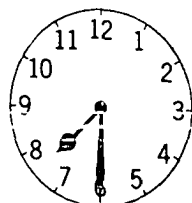
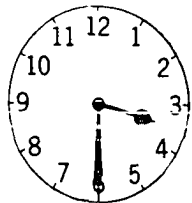
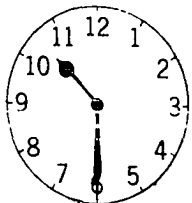
 CD is the longer segment.

 AB is the same length as CD.

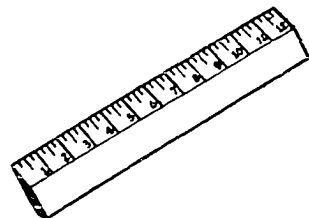
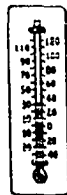
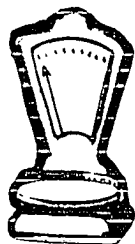
22



23



24



25

$$\square \quad 5 - 2 = \underline{\quad}$$

$$\square \quad 2 + 5 = \underline{\quad}$$

$$\square \quad 2 + 3 = \underline{\quad}$$

$$\square \quad 7 - 2 = \underline{\quad}$$

26

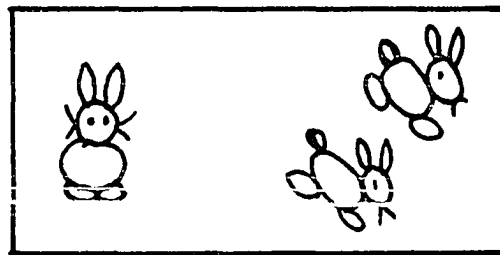
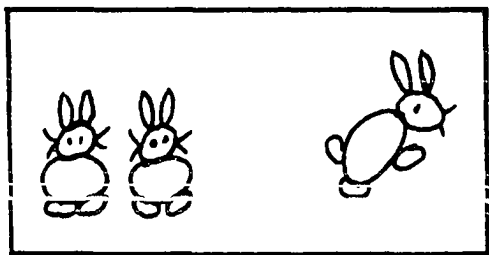
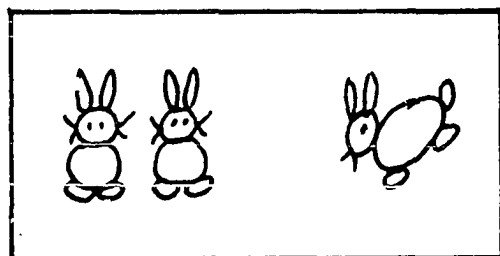
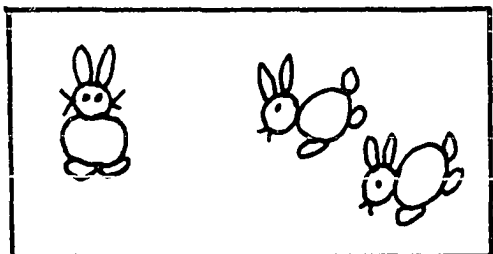
$$\square \quad 3 + 4 = 7$$

$$\square \quad 6 + 2 = 7$$

$$\square \quad 4 + 3 = 7$$

$$\square \quad 5 + 2 = 7$$

27



28 - 31



$$3 + 4 = \square$$

$$6 + 2 = \square$$

$$8 + 1 = \square$$

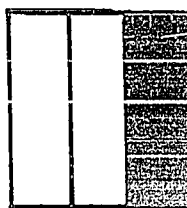
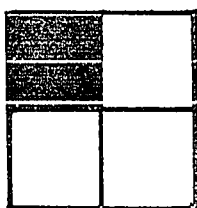
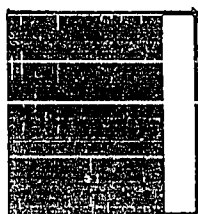
$$0 + 2 = \square$$

Part Two

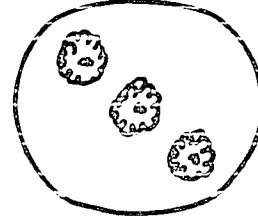
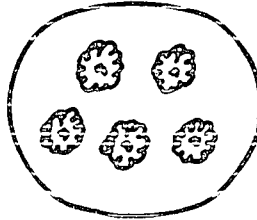
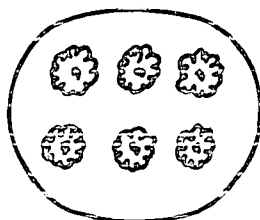
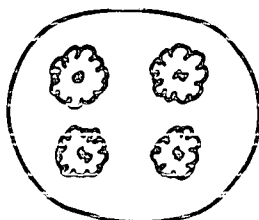
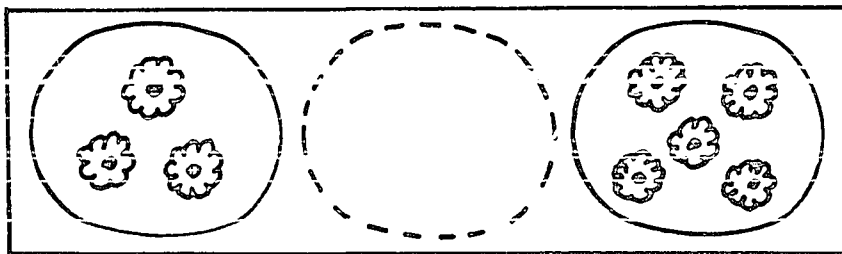
32



33



34



35

$8 - 2$

$6 + 3$

$10 - 2$

$8 + 1$

36

 four

 eight

 five

 three

37

12

14

16

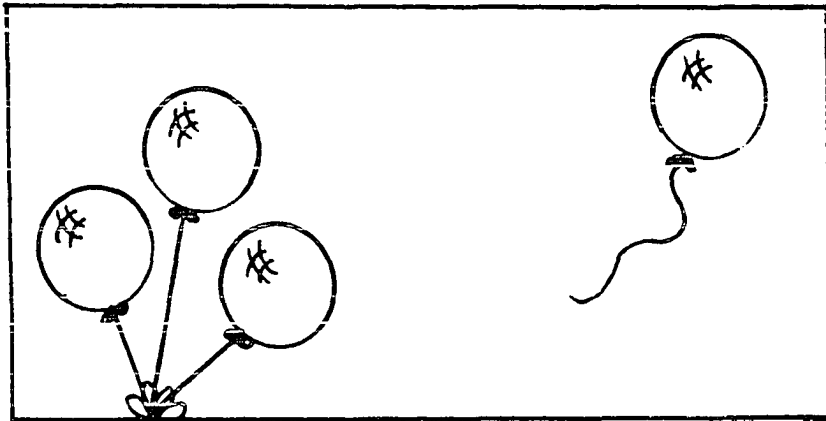
41

38

$3 + 5 \bigcirc 8$

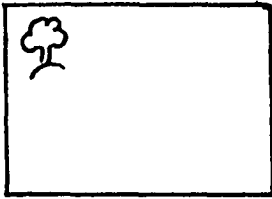
 $<$
 $-$
 $+$
 $=$

39

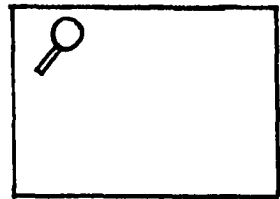

 Subtraction

 Addition

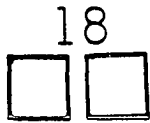
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41

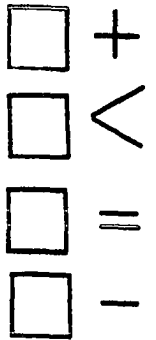


42

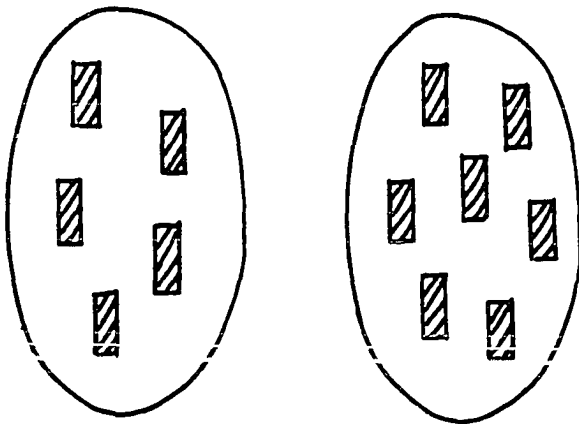


43

4 ○ 2 = 6

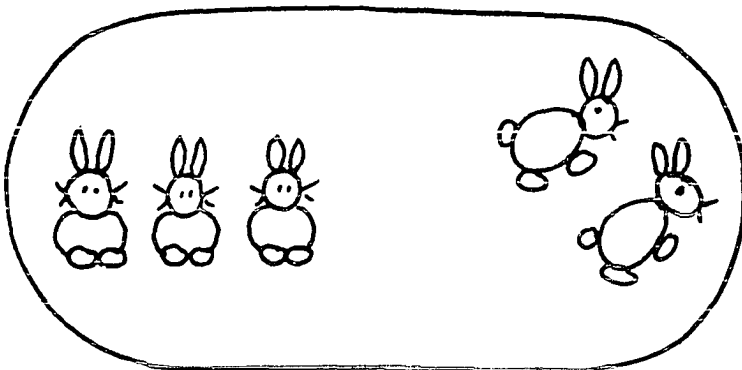


44



- 7 is less than 5.
- 7 is equal to 5.
- 7 is greater than 5

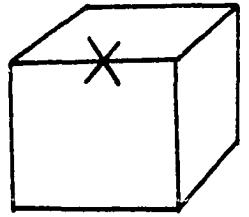
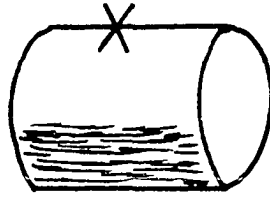
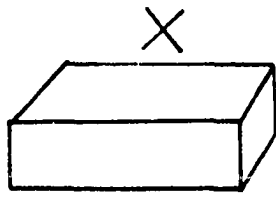
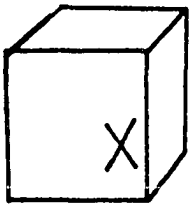
45



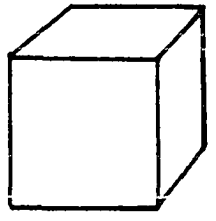
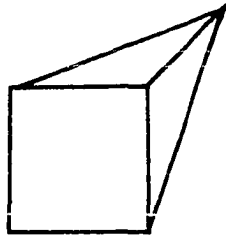
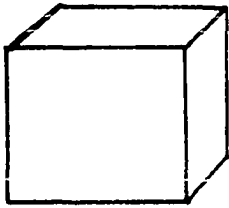
5 - 2 = 3



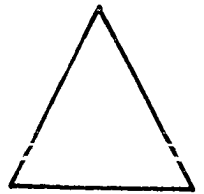
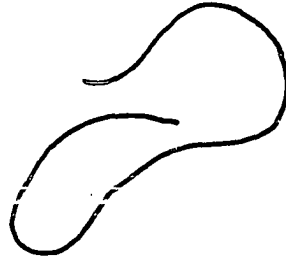
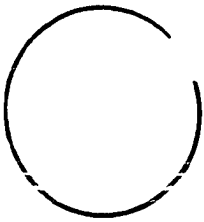
46



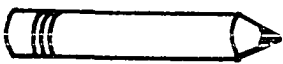
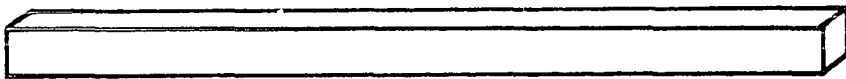
47



48



49



2



3



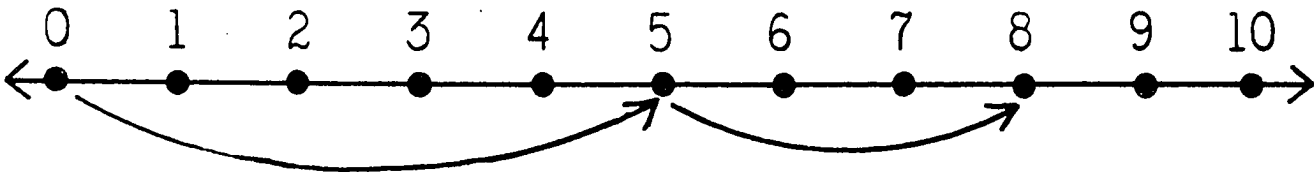
4



5

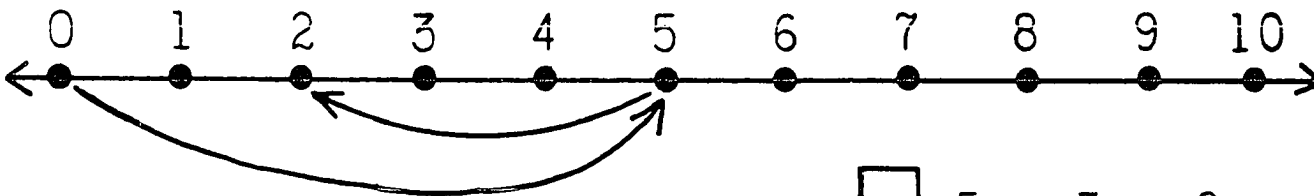


50



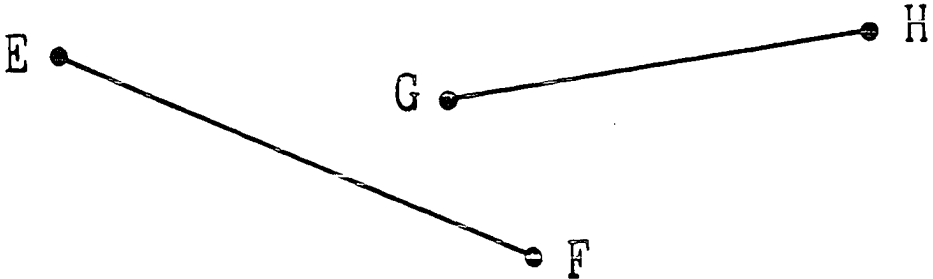
- $6 + 2 = 8$
 $5 + 3 = 8$
 $0 + 6 = 6$
 $5 + 2 = 7$

51



- $5 - 3 = 2$
 $5 - 1 = 4$
 $6 - 4 = 2$
 $5 - 2 = 3$

52

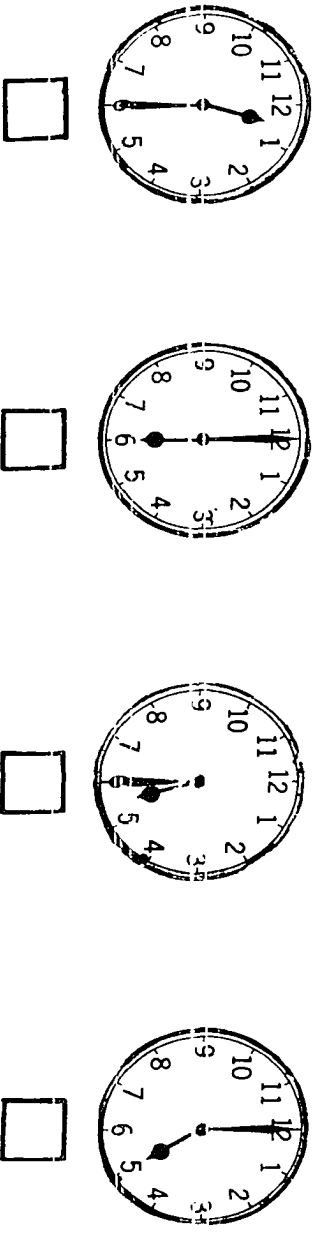


- EF is the shorter segment.
 GH is the shorter segment.
 EF is the same length as GH.

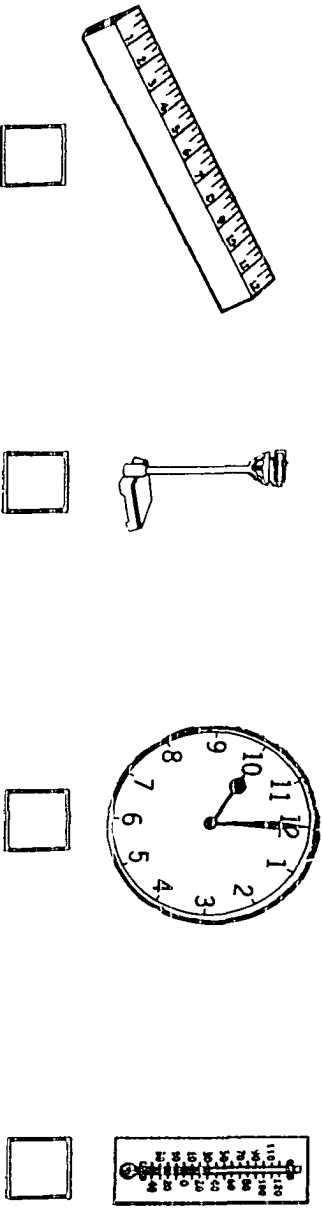
53



54



55



56

$$\square 5 + 2 = \underline{\quad}$$

$$\square 4 + 2 = \underline{\quad}$$

$$\square 4 - 2 = \underline{\quad}$$

$$\square 6 - 2 = \underline{\quad}$$

57

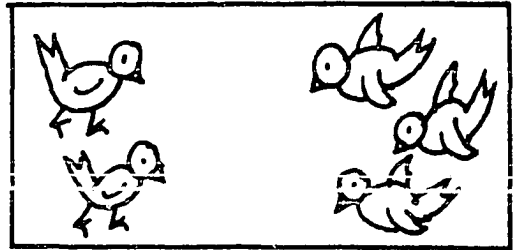
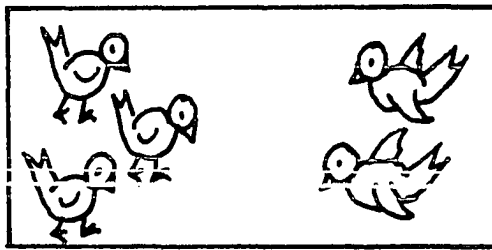
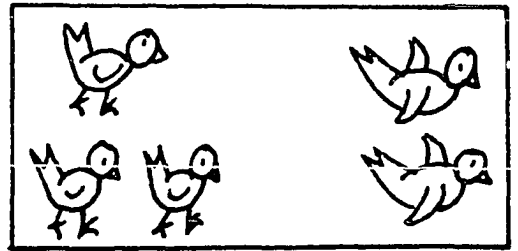
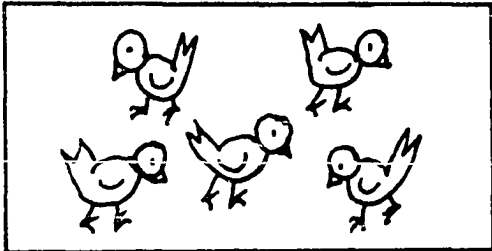
$$\square 6 + 3 = 9$$

$$\square 8 + 1 = 9$$

$$\square 7 + 3 = 9$$

$$\square 4 + 5 = 9$$

58



59 - 62



$$6 - 2 = \square$$

$$7 - 0 = \square$$

$$3 - 3 = \square$$

$$10 - 4 = \square$$

FIRST GRADE

ANSWER KEY

<u>PART ONE</u>				
<u>Page 1</u>	<u>Page 2</u>	<u>Page 3</u>	<u>Page 4</u>	<u>Page 5</u>
1 - B	4 - A	9 - 60	15 - C	19 - C
2 - D	5 - B	10 - A	16 - C	20 - D
3 - A	6 - C	11 - 0	17 - D	21 - A
	7 - D	12 - A	18 - C	
	8 - B	13 - B		
		14 -		
		6-2 =4 or 6-4 =2		
<u>PART TWO</u>				
<u>Page 6</u>	<u>Page 7</u>	<u>Page 8</u>	<u>Page 9</u>	<u>Page 10</u>
22 - D	25 - B	32 - C	35 - C	40 - 19
23 - B	26 - B	33 - B	36 - C	41 - 74
24 - A	27 - C	34 - A	37 - B	42 - B
	28 - 7		38 - D	43 - A
	29 - 8		39 - A	44 - C
	30 - 9			45 -
	31 - 2			3+2 =5 or 2+3 =5
<u>Page 11</u>	<u>Page 12</u>	<u>Page 13</u>	<u>Page 14</u>	
46 - D	50 - B	53 - A	56 - C	
47 - D	51 - A	54 - B	57 - C	
48 - D	52 - B	55 - D	58 - C	61 - 0
49 - B			59 - 4	62 - 6
			60 - 7	

APPENDIX

DIAGNOSTIC DATA

Following is the coding of items in the 1st grade test as they relate to the behavioral objectives in Wisconsin K-6 Mathematics Guidelines. The first numeral in the code relates to grade level. The second numeral relates to horizontal strand number (1-15). The strands are found on the fold-out pages. The third numeral is the sub-objective as numbered in the Guideline.

FIRST GRADE

<u>PROBLEM</u>	<u>CODE</u>	<u>PROBLEM</u>	<u>CODE</u>
1.....	1-01-1	31.....	1-06-1
2.....	1-01-2		1-15-2
3.....	1-01-5	32.....	1-01-3
4.....	1-02-2	33.....	1-01-2
5.....	1-02-1	34.....	1-01-5
6.....	1-02-4	35.....	1-02-2
7.....	1-04-3	36.....	1-02-3
8.....	1-04-1	37.....	1-02-4
9.....	1-02-1	38.....	1-04-5
10.....	1-02-4	39.....	1-04-2
11.....	1-01-4	40.....	1-02-1
12.....	1-04-5	41.....	1-02-1
13.....	1-03-1	42.....	1-02-4
14.....	1-04-4	43.....	1-04-5
15.....	1-07-1	44.....	1-03-2
	1-07-2	45.....	1-04-4
16.....	1-07-1	46.....	1-07-1
17.....	1-08-1		1-07-2
18.....	1-08-2	47.....	1-07-1
19.....	1-12-1	48.....	1-08-2
20.....	1-12-1	49.....	1-14-5
21.....	1-14-1	50.....	1-12-1
22.....	1-14-2	51.....	1-12-1
23.....	1-14-3	52.....	1-14-1
24.....	1-14-4	53.....	1-14-2
25.....	1-15-1	54.....	1-14-3
26.....	1-15-3	55.....	1-14-4
27.....	1-15-4	56.....	1-15-1
28.....	1-06-1	57.....	1-15-3
	1-15-2	58.....	1-15-4
29.....	1-06-1	59.....	1-06-1
	1-15-2		1-15-2
30.....	1-06-1		
	1-15-2		

<u>Objective</u>	<u>% of Sampled Students Responding Correctly</u>
1-01-1	79
1-01-2	60
1-01-3	91
1-01-4	76
1-01-5	58
1-02-1	81
1-02-2	65
1-02-3	85
1-02-4	74
1-03-1	83
1-03-2	80
1-04-1	63
1-04-2	77
1-04-3	82
1-04-4	33
1-04-5	74
1-06-1	75
1-07-1	65
1-07-2	68
1-08-1	61
1-08-2	68
1-12-1	52
1-14-1	57
1-14-2	66
1-14-3	73
1-14-4	90
1-14-5	55
1-15-1	67
1-15-2	75
1-15-3	66
1-15-4	64

ADDITIONAL TECHNICAL INFORMATION

During the processing of data obtained from the sample, discrimination indices were computed for all test items. They were averaged for each grade level, as follows:

Grade Level	Mean Discrimination Index
First	.46
Second	.43
Third	.42
Fourth	.33
Fifth	.35
Sixth	.34

Formula used to compute discrimination indices was

$$D. I. = \frac{N_h - N_l}{n}$$

Where D. I. is the discrimination index, N_h is the number in the "high group" (upper 27%) answering the item correctly, N_l is the number in the "low group" (lower 27%) answering the item correctly, and n is the total number in the high group (same as the total number in the low group).

Reliability measures were determined for the various grade level tests using the Kuder-Richardson Formula #20. Sample used was the one described previously. Results:

FIRST GRADE	.937
SECOND GRADE	.931
THIRD GRADE	.858
FOURTH GRADE	.799
FIFTH GRADE	.820
SIXTH GRADE	.792

VALIDITY

A survey conducted three years after publication of Guidelines to Mathematics, K-6 indicated that the mathematics program outlined in the guide is well accepted by Wisconsin Mathematics Educators. Sixty three per cent of the respondents rated the program "very good" and twenty five per cent rated it "excellent". This professional acceptance of the objectives in the guide enhances the validity of WMT. The writing committee unanimously agreed to declare content validity for the test because of its correlation with the objectives listed in the state guide.

In addition to the professional acceptance of the objectives, several independent studies made by university students have shown that most contemporary elementary mathematics textbook series contain the topics outlined in the state guide. School districts using up-to-date contemporary mathematics textbooks can therefore be assured that accepting the objectives in the guide as reasonable for their programs is logical, and that WMT is a valid instrument to use in their assessment activities.

In addition to correlating missed test items with objectives, which can be done using the above information, it is possible to compare results on a given objective within a district, school or class to broader results on the objective. The following charts show percentages of students responding correctly to items based on specific objectives. If more than one test item was included for a specific objective, the average of results for the several items was computed for the objective.

The selected stratified sample of Wisconsin students tested to obtain data for the following charts was determined as follows: First all Wisconsin public school districts were stratified according to size of city, size of district, socio-economic level, per-pupil expenditure for education, and per-pupil valuation of district. School districts then were drawn at random from three categories, under 1000 elementary students, between 1000 and 5000 elementary students, and "metropolitan". Thus a list of 13 districts was obtained including 5 in the first category, 4 in the second category, and 4 in the third category. A list of alternates was similarly established, 8 in the first category, 7 in the second, and 5 in the third. Writing committee members then arbitrarily selected districts in 13 different geographical areas of the state and made arrangements for tests to be given (each grade level) in 8 Milwaukee classrooms, 9 other "metropolitan" classrooms (three per district), 12 classrooms in districts with from 1000 to 5000 elementary students, and 10 classrooms in districts having less than 1000 elementary students. Approximately 1000 students were given each test.

APPENDIX D.

ELMR-2

Directions for Administering Test

(To be administered in two sittings separated by a recess period.)

MATERIALS

Test booklet
Soft lead pencil
Eraser

FRONT PAGE


I am going to give each of you a booklet with some arithmetic problems for you to do. Do not open your booklet until I tell you to do so.

PASS OUT THE TEST BOOKLETS

Look at the front page. Find the star and the word Name. Write your first name and last name on the line after the word Name.

SAMPLE A

We are going to do some arithmetic problems together.

(Have Sample A on the chalkboard )

Put a cross in the box under the circle.
(Show how to mark with the sample on the chalkboard.)
(Check to see that each child follows directions for marking.)

SAMPLE B

Look at the numerals in row B. Put a cross in the box under the numeral 8.

(Repeat) The numeral 8.

SAMPLE C

Look across the page at the next problem, problem C. Look at the number word. Put a cross in the box beside the numeral that is the same as the number word. (Repeat directions)

(Check to see that each child follows directions.)

NOW TURN THE PAGE

Look at the top of the page where it says, "Sample." Put your finger on the word "Sample." The number sentence in the box says: "2 + 5 is what?" Now look at the letters and numbers under the number sentence. A - 5, B - 6, C - 7, D - 8. Which of the numbers will make the number sentence true? Yes, it is 7, so put a cross on the letter in front of the number seven, because this is the correct answer. We will mark all of our answers in this way.

I will read the directions and you will put a cross on the letter in front of the correct answer.

Listen carefully.

PAGE 1

Behavioral Objectives

- | | |
|---|-----------|
| 1. Find the numeral four hundred five. Mark the letter in front of the numeral four hundred five. | 2 - 2 - 1 |
| 2. Which of the expanded numerals is four hundred nine? Mark the letter in front of | 2 - 2 - 5 |

PAGE 1 (continued)Behavioral
Objectives

the expanded numeral for four hundred nine.

- | | | |
|----|--|-----------|
| 3. | Look at the box with the tree in it. Write the numeral seven hundred sixty in this box. | 2 - 2 - 1 |
| 4. | Look at the boxes with the numerals in them. Complete the number pattern. | 2 - 2 - 3 |
| 5. | Look at the box. See the numbers, 6 + 3, 5 + 4, 7 + 2. Find another name for these numbers. Mark the letter in front of another name for the numbers in the box. | 2 - 2 - 2 |
| 6. | Which of the following is the order of counting numbers? Mark the letter in front of the numbers in the order of counting numbers. | 2 - 3 - 5 |
| 7. | Which of the following number sentences is true? Mark it. | 2 - 4 - 6 |

TURN THE PAGE

PAGE 2

- | | | |
|-----|---|-----------|
| 8. | What number comes between 49 and 51? | 2 - 3 - 3 |
| 9. | What numbers come just before and just after 56? | 2 - 3 - 4 |
| 10. | Look at the number sentence in the box. Find the number which will make the sentence true. Mark it. | 2 - 6 - 3 |

PAGE 2 (continued)

- | | <u>Behavioral Objectives</u> |
|---|------------------------------|
| 11. Look at the number sentence in the box. Mark the letter in front of the symbol which will make the sentence true. | 2 - 15 - 4 |
| 12. Which of the following addition problems is true? Mark it. | 2 - 6 - 2 |
| 13. Look at the number sentence in the box. Find the picture that goes with the number sentence. Mark it. | 2 - 15 - 2 |
| 14. Look at the circle. The shaded part of the circle is: | |
| A. one half of the circle. | |
| B. one fourth of the circle. | |
| C. one third of the circle. | |
| D. two thirds of the circle. | |
| Mark the letter in front of the true statement. | 2 - 1 - 3 |

TURN THE PAGE

PAGE 3

- | | |
|--|-----------|
| 15. Look at the pictures of the sticks. Mark the letter of the picture which shows three hundred forty two sticks. | 2 - 2 - 4 |
| 16. Find the sentence which is true. Mark it. | 2 - 3 - 1 |

PAGE 3 (continued)Behavioral Objectives

17. Look at the number sentence.
Mark the letter in front of the sign which is missing from the number sentence. 2 - 3 - 2

TURN THE PAGE

PAGE 4

18. Look at the picture of sticks in the box. Find another picture which shows the same number of sticks as the sticks in the box. Mark it. 2 - 2 - 6
19. Which of the following statements is true? Mark the letter in front of the statement which is true. 2 - 4 - 1
- A. Eight hundred sixty four million is the largest number there is.
- B. One hundred ninety nine million, nine hundred ninety nine thousand, nine hundred ninety nine is the largest number there is.
- C. One trillion is the largest number there is.
- D. There is no largest number.
20. Look at the figures. Which figure is a cylinder? What is the number of that figure? Find the numbers below

PAGE 4 (continued)Behavioral
Objectives

the figures and mark the letter
in front of the number of the 2 - 7 - 1
cylinder.

TURN THE PAGE

PAGE 5

21. The black duck is leading the ducks
to the pond. Which duck in line
has an X on it? Mark the letter 2 - 1 - 2
in front of the word that tells us.
22. Look at the picture of the ruler.
The numbered units are inches. 2 - 14 - 1
The arrow points towards:
- A. a unit.
 - B. a half unit.
 - C. a third unit.
 - D. an eighth unit.
- Mark the statement which makes the
sentence true.
23. In the picture of the measuring tools,
what is the numeral under the
measuring tool you would use to 2 - 14 - 3
measure the length of a pencil.
Mark the letter in front of that
numeral.
24. What is the time shown on the clock?
Find it and mark it. 2 - 14 - 4
25. Look at the numeral in the box.
What number is $60 + 12$? 2 - 15 - 5

PAGE 5 (continued)Behavioral Objectives

- 26 - Look at each number
27. sentence. A number is missing. Mark the letter in front of the numeral which will make the number sentence true.
- 2 - 15 - 1
and
2 - 6 - 1

TURN THE PAGE

PAGE 6

- 28., 29., 30., 31. Look at each number sentence. A number is missing. Mark the letter in front of the numeral which will make the number sentence true. Be sure to watch the signs. Some number sentences are addition sentences and some are subtraction sentences.
- 2 - 15 - 1
and
2 - 6 - 1
32. Which of the following statements is not true? Mark the letter in front of that statement. 2 - 4 - 3
33. Look at the figure. 2 - 8 - 1

Point A:

- A. stands for a line.
B. stands for a line segment.
C. stands for an angle.
D. stands for a place on the line.

Mark the letter in front of the statement which finishes the sentence correctly.

34. Look at the line segments.

PAGE 6 (continued)Behavioral
Objectives

Which of the line segments appears to be congruent? Mark the letter 2 - 10 - 2 in front of the names of the line segments which appear to be congruent.

TURN THE PAGE

PAGE 7

35. Look at the picture. This figure is: 2 - 8 - 3
- A. a simple curve.
 - B. a line segment.
 - C. a simple closed curve.
 - D. a polygon.

Mark the statement which is true.

36. Look at the figure. Listen carefully.
- A. The square is outside the circle.
 - B. The circle is outside the triangle.
 - C. The triangle is outside the square. 2 - 8 - 5
 - D. The triangle is outside the circle.

Mark the statement which is true.

37. Look at the number line. This number line shows counting by: 2 - 12 - 1
- A. 10's
 - B. 6's
 - C. 2's
 - D. 5's

Mark the one which is true.

Behavioral
Objectives

Close your book. - Collect books.

Second Sitting

PART II

Today we are going to play some other arithmetic games. Open your book to page 8. Look at example number 38.

38. Find the numeral eight hundred seventy. Mark the letter in front of the numeral eight hundred seventy. 2 - 2 - 1
39. Look at the box with the flower in it. Write the numeral nine hundred ninety nine in this box. 2 - 2 - 1
40. Look at the boxes with the numerals in them. Complete the number pattern. 2 - 2 - 3
41. Look at the box. See the numbers, 5 - 4, 3 - 2, 4 - 3. Find another name for these numbers. Mark the letter in front of another name for the numbers in the box. 2 - 2 - 2
42. What number comes between 208 and 210? 2 - 3 - 3
43. What numbers come just before and just after 89? 2 - 3 - 4
44. Look at the number sentence. Mark the letter in front of the sign which is missing from the number sentence. 2 - 3 - 2

TURN THE PAGE

45. Listen carefully.

Jose has 25¢. He bought
17¢ worth of candy. How
much change does he get?

2 - 14 - 5

Mark the letter in front of the
change Jose will get.

46. Listen carefully. I will read a
story problem.

2 - 14 - 2

Mary finds she can serve
3 of her friends from a
pint of ice cream. How
many can she serve from
a quart of ice cream?

Mark the letter in front of the
number which tells how many
friends she can serve from a
quart of ice cream.

47. On the scale shown, how much
does the box weigh? Mark it.

2 - 14 - 2

48 - Look at each number sentence.

2 - 15 - 1

49. A number is missing. Mark
the letter in front of the
numeral which will make the
number sentence true. Be
sure to watch the signs.

and
2 - 6 - 1

TURN THE PAGE

Page 10

50., 51., 52., 53. Look at each
number sentence. A number
is missing. Mark the letter
in front of the numeral which
will make the number sentence
true. Be sure to watch the
signs. Some number sentences

2 - 15 - 1
and
2 - 6 - 1

Behavioral
Objectives

are addition sentences and
some are subtraction sentences.

54. Which one of the numbers makes
the sentence true? 2 - 4 - 2
55. Look at the number sentence.
Mark the letter in front of the
symbol which will make the
sentence true. 2 - 15 - 4
56. Look at the number sentence
in the box. You may use the
number line to find the number
which will make the sentence
true. Mark it. 2 - 15 - 3
57. Which of the following number
sentences is true? Mark it. 2 - 4 - 6

TURN THE PAGE

PAGE 11

58. The picture shows that three
sets of four objects are joined
together to make a new set.
Mark the letter in front of
the word that tells what kind
of number sentence you would
write for this picture. 2 - 4 - 4
- A. Subtraction
B. Division
C. Multiplication
59. Look at the number sentence in
the box. Find the number which
will make the sentence true.
Mark it. 2 - 6 - 3

PAGE 11 (continued)

	<u>Behavioral Objectives</u>
60. Look at the number sentence in the box. Find the number which will make the sentence true. Mark it.	2 - 6 - 3
61. Which number is the same as fifty nine? Mark the letter in front of the number which is the same as fifty nine.	2 - 15 - 5
62. Look at the number sentence in the box. Find the picture that goes with the number sentence. Mark it.	2 - 15 - 2
63. Look at the number sentence. Mark the letter in front of the sign which is missing from the number sentence.	2 - 3 - 2
64. The clown with the balloon is the leader. Which clown has a line around him? Mark the letter in front of the word that tells us.	2 - 1 - 2

TURN THE PAGE

PAGE 12

65. Look at the addition table. Find the squares with the missing numbers. What pattern is formed by the missing numbers?	2 - 4 - 5
---	-----------

The pattern formed is:

- A. 5, 7, 9, 11, 13, 15.
- B. 3, 6, 9, 12, 15, 18.
- C. 4, 6, 8, 10, 12, 14.
- D. 3, 4, 5, 6, 7, 8.

PAGE 12 (continued)Behavioral Objectives

66. Look at the square. The shaded part of the square is: 2 - 1 - 3
- A. one half of the square.
 - B. one third of the square.
 - C. three thirds of the square.
 - D. one fourth of the square.

Mark the letter in front of the true statement.

TURN THE PAGE

PAGE 13

67. Look at the picture of sticks in the box. Find another picture 2 - 2 - 6 which shows the same number of sticks as the sticks in the box. Mark it.
68. Look at the picture of the sticks. Find the numeral that tells how 2 - 1 - 1 many sticks there are. Mark it.

TURN THE PAGE

PAGE 14

69. Look at the figures. Which one is a closed curve? Mark the letter under the closed curve. 2 - 8 - 4
70. Look at the three figures. Listen carefully. 2 - 8 - 5
- A. The circle is inside the square.
 - B. The square is inside the triangle.
 - C. The square is inside the

Behavioral
Objectives

circle.

- D. The circle is inside the triangle.

Mark the statement which is true.

71. Look at the picture of the line segment and the ruler. The line segment AB is: 2 - 14 - 2
- A. 4 1/2 inches long.
 B. 4 inches long.
 C. 3 inches long.
 D. 2 inches long.

Mark the statement which is true.

TURN THE PAGE

PAGE 15

72. Look at the line segment. Mark 2 - 8 - 1
 the statement which is true.

This line segment

- A. has no points.
 B. is a set of many points.
 C. is a set of 2 points.
 D. is a set of 400 points.
73. Look at the picture of the line. 2 - 8 - 2
 This straight line:
- A. is a set of points which stops where the drawing of the line ends.
 B. is a set of points which stops at the end of this paper.
 C. has end points.
 D. is a set of points which never ends.

Behavioral
Objectives

Mark the statement which
is true.

74. Look at the triangles. Which
triangles appear to be con-
gruent? Mark the letter in
front of the numbers which
tell which triangles appear to
be congruent. 2 - 10 - 1

MATHEMATICS TEST
SECOND GRADE

Name _____ last _____ first _____

Boy _____ Girl _____ Grade _____

School _____ Teacher _____

City _____ State _____

Date of Testing _____
year month day

Date of Birth _____
year month day

Age _____
years months

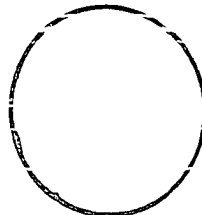
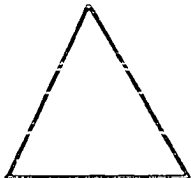
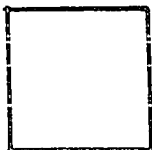
TEST SCORES	
Number correct	_____
Number wrong	_____
Number omitted	_____
Total test items - 74	



Name _____

Samples

A



B

6



7



8



9



C



0



1



2



3

two

Second Grade - Part I

Sample :

$2 + 5 =$
--

- A. 5
B. 6
C. 7
D. 8

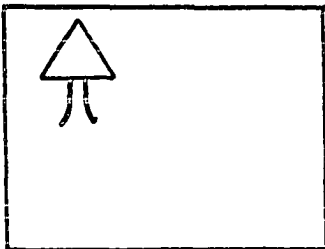
1.

- A. 4050
B. 405
C. 4005
D. 40050

2.

- A. $400 + 90 + 0$
B. $900 + 40 + 0$
C. $400 + 40 + 9$
D. $400 + 0 + 9$

3.



4.

15	20	25			
----	----	----	--	--	--

5.

$6 + 3, 5 + 4, 7 + 2 \dots$

- A. $9 + 1$
B. $1 + 8$
C. $7 + 3$
D. $5 + 6$

6.

- A. 58, 59, 60, 61
B. 59, 61, 63, 62
C. 73, 70, 71, 72
D. 72, 74, 73, 76

7.

- A. $7 - 0 = 0$
B. $7 - 6 = 0$
C. $7 - 7 = 0$
D. $7 - 8 = 0$

8.

49, _____, 51

- A. 48
- B. 52
- C. 50
- D. 47

12. 273

A.	B.	C.	D.
7	4	8	9
1	3	6	0
2	9	2	4
-	-	-	-
12	16	20	14

9.

_____, 56, _____

- A. 55 and 57
- B. 55 and 58
- C. 53 and 56
- D. 54 and 57

13.

- 7 = 5

- A.
XXX
XXX
XXX
XXX
- B.
XXXX
XXXX
XXXX
- C.
XXX
XXX
XXXX
XXXX
- D.
XXXXXX
XXXXXX
XXXXXX

10.

4 X 3 =

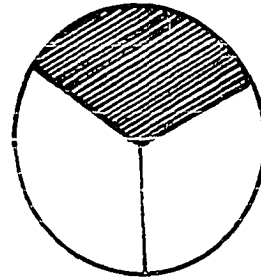
- A. 12
- B. 7
- C. 43
- D. 16

11.

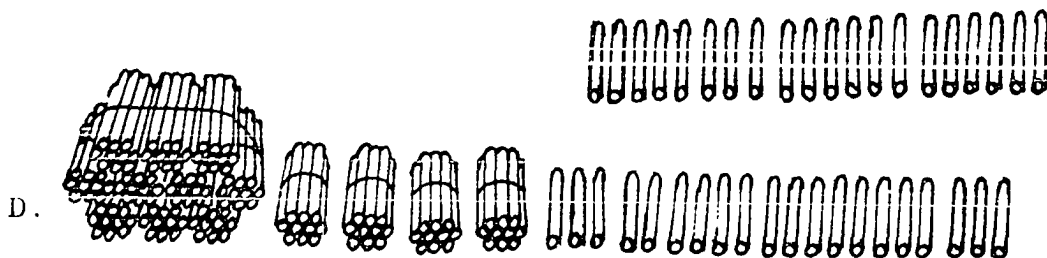
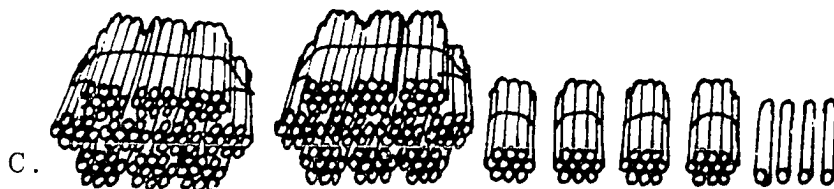
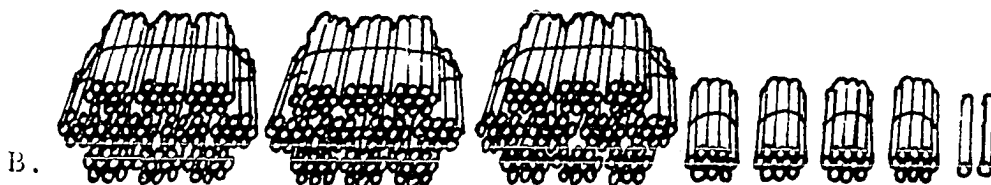
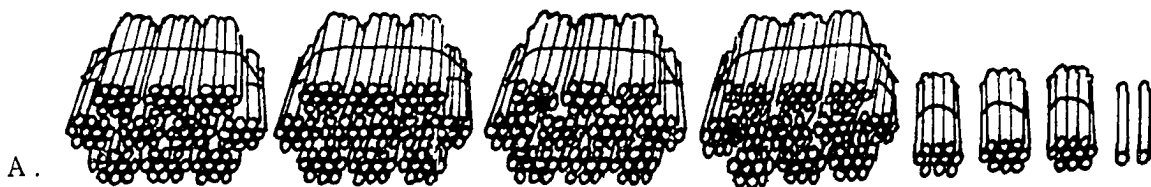
4 3 = 7

- A. +
- B. -
- C. <
- D. X

14.



- A. one half of the circle.
- B. one fourth of the circle.
- C. one third of the circle.
- D. two thirds of the circle.



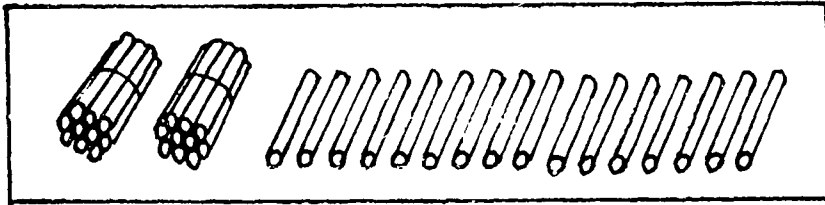
16.

- A. 5 is greater than 7.
- B. 7 is less than 8.
- C. 7 equals 6.
- D. 9 is greater than 10.

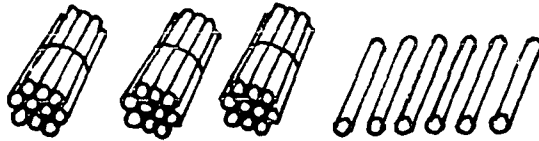
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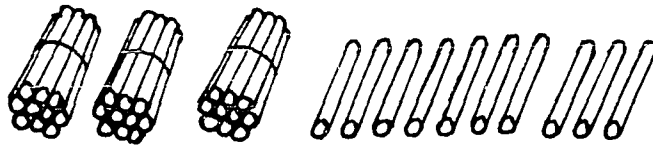
- A. =
- B. <
- C. X
- D. >



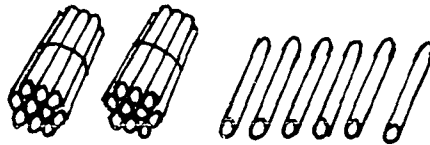
A.



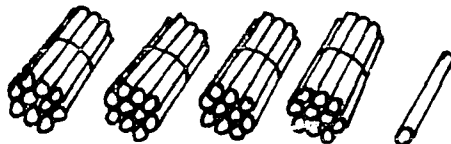
B.



C.



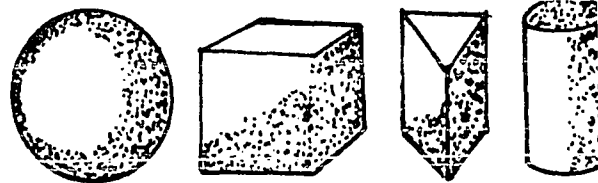
D.



19.

- A. 864,000,000 is the largest number there is.
- B. 199,999,999 is the largest number there is.
- C. One trillion is the largest number there is.
- D. There is no largest number.

20.



1

2

3

4

- A. 1
- B. 2
- C. 3
- D. 4

279
Mathematics Test

for

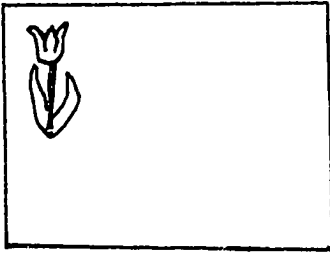
Behavioral Objectives

Second Grade - Part II

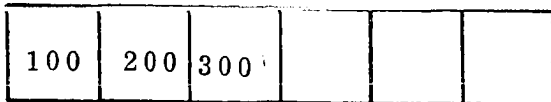
38.

- A. 80070
- B. 8070
- C. 870
- D. 807

39.



40.



41.

5 - 4, 3 - 2, 4 - 3...

- A. 2 - 2
- B. 2 - 1
- C. 4 - 2
- D. 5 - 1

42.

208, _____, 210


- A. 207
- B. 211
- C. 206
- D. 209

43.

_____, 89, _____

- A. 86 and 90
- B. 86 and 91
- C. 88 and 91
- D. 88 and 90

44.

62  52

- A. =
- B. <
- C. X
- D. >

28. $\square + 6 = 11$

- A. 3
- B. 4
- C. 5
- D. 6

29. $\square - 8 = 8$

- A. 16
- B. 17
- C. 18
- D. 19

30. $17 - \square = 8$

- A. 7
- B. 9
- C. 4
- D. 5

31. $12 - 8 = \square$

- A. 7
- B. 6
- C. 5
- D. 4

32. ²⁷⁷

- A. If $3 + 4 = 7$ then
 $4 + 3 = 7$
- B. If $7 - 4 = 3$ then
 $7 - 3 = 4$
- C. If $7 - 3 = 4$ then
 $3 - 7 = 4$
- D. If $2 + 5 = 7$ then
 $5 + 2 = 7$

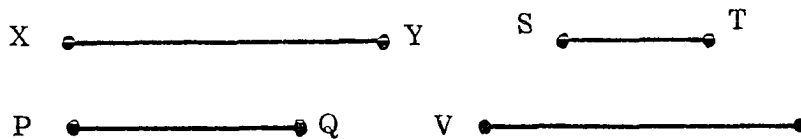
33.



Point A

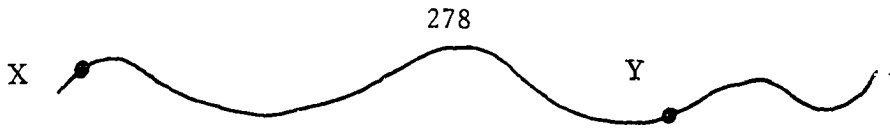
- A. stands for a line.
- B. stands for a line segment.
- C. stands for an angle.
- D. stands for a place on the line.

34.



- A. PQ and XY
- B. PQ and ST
- C. ST and VW
- D. XY and VW

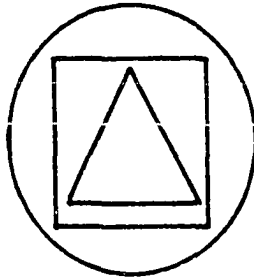
35.



This figure is

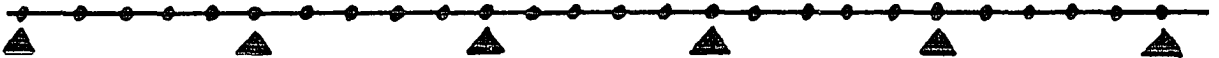
- A. a simple curve.
 - B. a line segment.
 - C. a simple closed curve.
 - D. a polygon.
-

36.



- A. The square is outside the circle.
 - B. The circle is outside the triangle.
 - C. The triangle is outside the square.
 - D. The triangle is outside the circle.
-

37.



This number line shows counting by

- A. 10's
- B. 6's
- C. 2's
- D. 5's

279
Mathematics Test

for

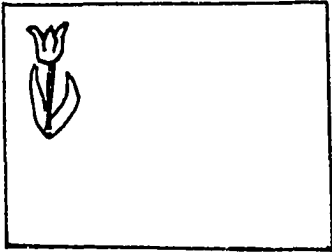
Behavioral Objectives

Second Grade - Part II

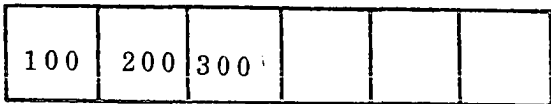
38.

- A. 80070
- B. 8070
- C. 870
- D. 807

39.



40.



41.

5 - 4, 3 - 2, 4 - 3...

- A. 2 - 2
- B. 2 - 1
- C. 4 - 2
- D. 5 - 1

42.

208, _____, 210

- A. 207
- B. 211
- C. 206
- D. 209

43.

_____, 89, _____

- A. 86 and 90
- B. 86 and 91
- C. 88 and 91
- D. 88 and 90

44.

62  52

- A. =
- B. <
- C. X
- D. >

45.

Jose had 25¢. He bought 17¢ worth of candy. His change was:

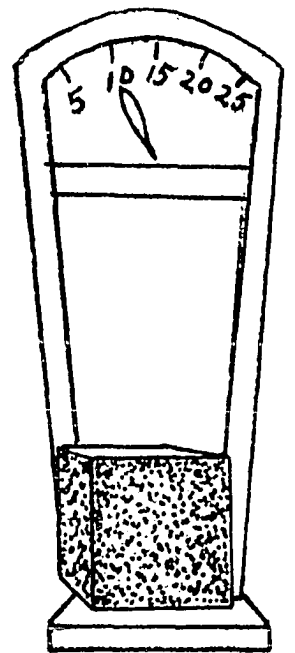


46.

Mary finds she can serve 3 of her friends from a pint of ice cream. How many can she serve from a quart of ice cream?

- A. 5
- B. 4
- C. 7
- D. 6

47.



- A. 10 pounds
- B. 24 pounds
- C. 1 pound
- D. 5 pounds

48.

$$4 + 9 = \square$$

- A. 10
- B. 11
- C. 12
- D. 13

49.

$$7 + \square = 15$$

- A. 7
- B. 8
- C. 9
- D. 10

50.

$$\square + 9 = 12$$

- A. 3
B. 4
C. 5
D. 6

51.

$$13 - \square = 6$$

- A. 5
B. 6
C. 7
D. 8

52.

$$\square - 8 = 0$$

- A. 1
B. 8
C. 0
D. 7

53.

$$18 - 9 = \square$$

- A. 6
B. 7
C. 8
E. 9

54.

$$3 + (4 + 5) = (3 + 4) + \square$$

- A. 7
B. 3
C. 4
D. 5

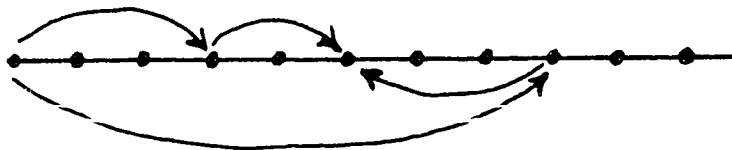
55.

$$6 \square 3 < 5$$

- A. +
B. -
C. <
D. X

56.

$$3 + 2 = 8 - \square$$

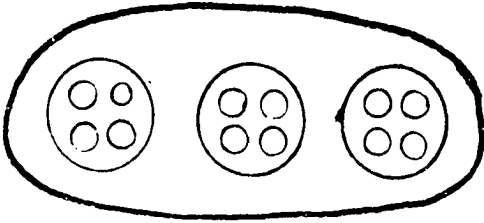


- A. 2
B. 3
C. 4
D. 5

57.

- A. $8 + 0 = 9$
B. $8 + 0 = 8$
C. $8 + 0 = 0$
D. $0 + 8 = 0$

58.



- A. Subtraction
- B. Division
- C. Multiplication

282

62.

$$4 + \square = 10$$

- A. $\begin{array}{ll} XX & 00 \\ XX & 00 \end{array}$
- B. $\begin{array}{ll} XX & 00 \\ XX & 00 \\ XX & 00 \end{array}$
- C. $\begin{array}{ll} XX & 00 \\ XX & 00 \end{array}$
- D. $\begin{array}{ll} XX & \checkmark \checkmark & 0 & 0 \\ XX & \checkmark & & 0 \end{array}$

59.

$$8 \times \square = \square$$

- A. 81
- B. 7
- C. 8
- D. 9

63.

$$3 + 2 \quad \text{O} \quad 2 + 3$$

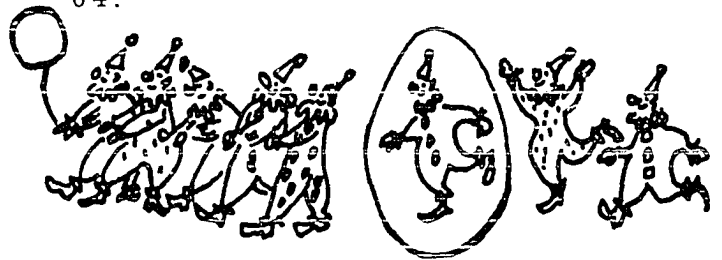
- A. =
- B. <
- C. X
- D. >

60.

$$3 \times \square = 18$$

- A. 9
- B. 16
- C. 6
- D. 3

64.



- A. third
- B. tenth
- C. seventh
- D. sixth

61.

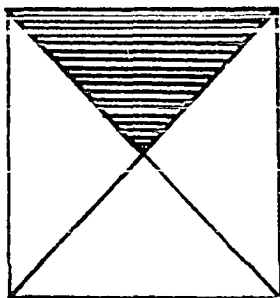
- A. $50 + 9$
- B. $5 + 9$
- C. $5 + 90$
- D. $90 + 5$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4		5	6	7	8	9	10	11	12	13
5	5		7	8	9	10	11	12	13	14
6	6	7		9	10	11	12	13	14	15
7	7	8	9		11	12	13	14	15	16
8	8	9	10	11		13	14	15	16	17
9	9	10	11	12	13		15	16	17	18

The pattern formed is:

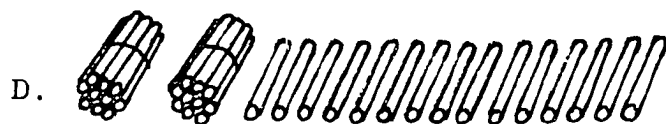
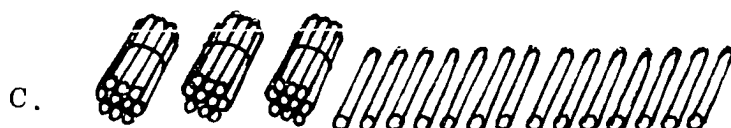
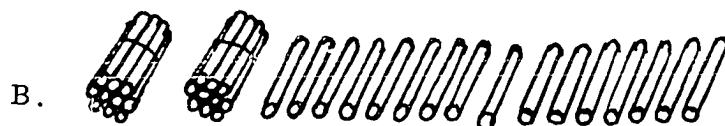
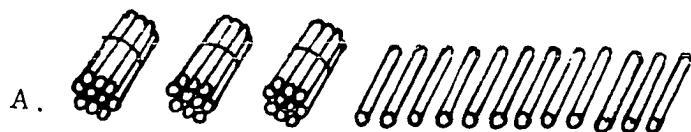
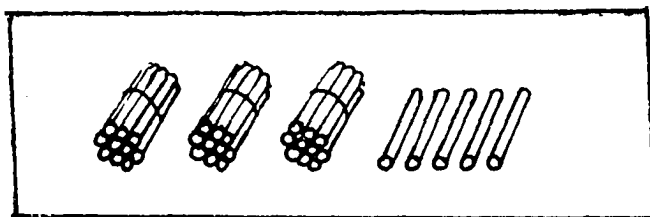
- A. 5, 7, 9, 11, 13, 15.
- B. 3, 6, 9, 12, 15, 18.
- C. 4, 6, 8, 10, 12, 14.
- D. 3, 4, 5, 6, 7, 8.

66.

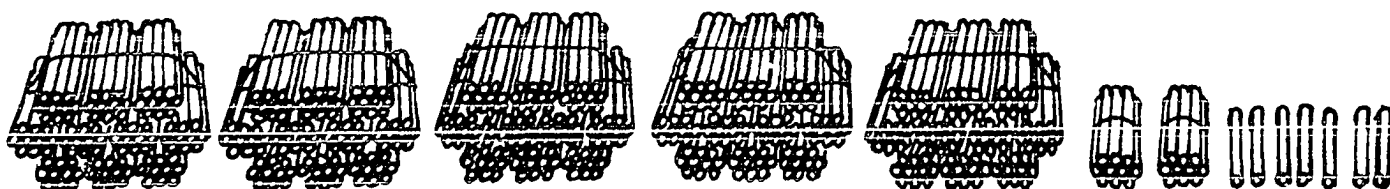


- A. one half of the square.
- B. one third of the square.
- C. three thirds of the square.
- D. one fourth of the square.

67.



68.



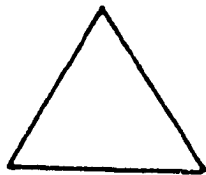
A. 752

B. 527

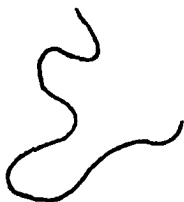
C. 725

D. 257

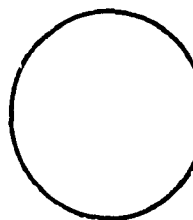
69.



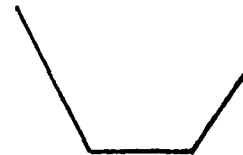
A.



B.

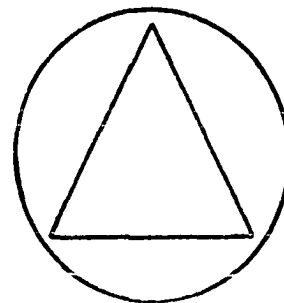
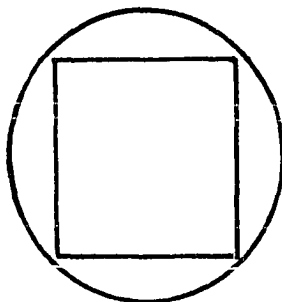
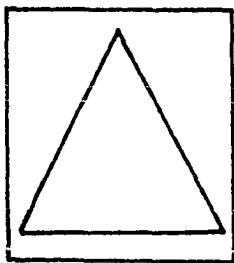


C.



D.

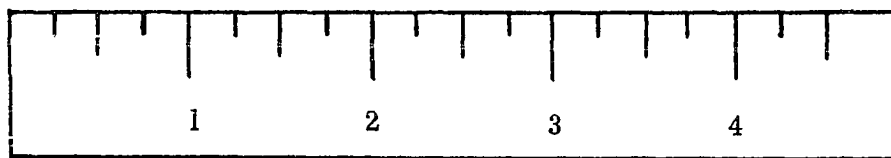
70.



- A. The circle is inside the square.
- B. The square is inside the triangle.
- C. The square is inside the circle.
- D. The circle is inside the triangle.

71.

A  B



Line segment AB above is

- A. $4 \frac{1}{2}$ inches long.
- B. 4 inches long.
- C. 3 inches long.
- D. 2 inches long.

This line segment

- A. has no points.
 - B. is a set of many points.
 - C. is a set of 2 points.
 - D. is a set of 400 points.
-

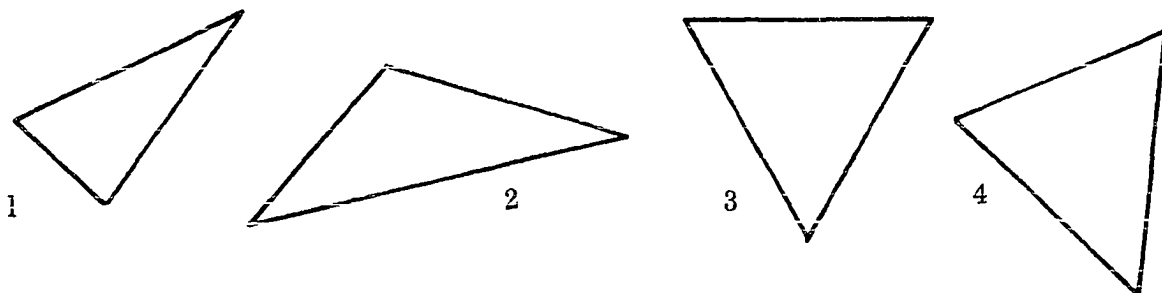
73.



This straight line

- A. is a set of points which stops where the drawing of the line ends.
 - B. is a set of points which stops at the end of this paper.
 - C. has end points.
 - D. is a set of points which never ends.
-

74.



- A. 2 and 1
- B. 2 and 3
- C. 3 and 4
- D. 4 and 1

287
SECOND GRADE
ANSWER KEY

- | | | |
|---------------|-------------------|-------|
| 1. B | 26. B | 51. C |
| 2. D | 27. A | 52. B |
| 3. 760 | 28. C | 53. D |
| 4. 30, 35, 40 | 29. A | 54. D |
| 5. B | 30. B | 55. B |
| 6. A | 31. D | 56. B |
| 7. C | 32. C | 57. B |
| 8. C | 33. D | 58. C |
| 9. A | 34. D | 59. C |
| 10. A | 35. A | 60. C |
| 11. A | 36. B | 61. A |
| 12. B | 37. D | 62. C |
| 13. A | 38. C | 63. A |
| 14. C | 39. 999 | 64. D |
| 15. B | 40. 400, 500, 600 | 65. C |
| 16. B | 41. B | 66. D |
| 17. B | 42. D | 67. D |
| 18. A | 43. D | 68. B |
| 19. D | 44. D | 69. A |
| 20. D | 45. A | 70. C |
| 21. C | 46. D | 71. D |
| 22. B | 47. A | 72. B |
| 23. C | 48. D | 73. D |
| 24. C | 49. B | 74. C |
| 25. D | 50. A | |

Following is the coding of items in the 2nd grade test as they relate to the behavioral objectives in Wisconsin K-6 Mathematics Guidelines.

The first numeral in the code relates to grade level. The second numeral relates to horizontal strand number (1-15). The strands are found on the fold-out pages. The third numeral is the sub-objective as numbered in the Guideline.

SECOND GRADE

<u>PROBLEM</u>	<u>CODE</u>	<u>PROBLEM</u>	<u>CODE</u>	<u>PROBLEM</u>	<u>CODE</u>
1.....	2-02-1	28.....	2-15-1	50.....	2-06-1
2.....	2-02-5		2-06-1		2-15-1
3.....	2-02-1	29.....	2-15-1	51.....	2-06-1
4.....	2-02-3		2-06-1		2-15-1
5.....	2-02-2	30.....	2-15-1	52.....	2-06-1
6.....	2-03-5		2-06-1		2-15-1
7.....	2-04-6	31.....	2-15-1	53.....	2-06-1
8.....	2-03-3		2-06-1		2-15-1
9.....	2-03-4	32.....	2-04-3	54.....	2-04-2
10.....	2-06-3	33.....	2-08-1	55.....	2-15-4
11.....	2-15-4	34.....	2-10-2	56.....	2-15-3
12.....	2-06-2	35.....	2-08-3	57.....	2-04-6
13.....	2-15-2	36.....	2-08-5	58.....	2-04-4
14.....	2-01-3	37.....	2-12-1	59.....	2-06-3
15.....	2-02-4	38.....	2-02-1	60.....	2-06-3
16.....	2-03-1	39.....	2-02-1	61.....	2-15-5
17.....	2-03-2	40.....	2-02-3	62.....	2-15-2
18.....	2-02-6	41.....	2-02-2	63.....	2-03-2
19.....	2-04-1	42.....	2-03-3	64.....	2-01-2
20.....	2-07-1	43.....	2-03-4	65.....	2-04-5
21.....	2-01-2	44.....	2-03-2	66.....	2-01-3
22.....	2-14-1	45.....	2-14-5	67.....	2-02-6
23.....	2-14-3	46.....	2-14-2	68.....	2-01-1
24.....	2-14-4	47.....	2-14-2	69.....	2-08-4
25.....	2-15-5	48.....	2-06-1	70.....	2-08-5
26.....	2-15-1		2-15-1	71.....	2-14-2
	2-06-1	49.....	2-06-1	72.....	2-08-1
27.....	2-15-1		2-15-1	73.....	2-08-2
	2-06-1			74.....	2-10-1

SECOND GRADE

<u>Objective</u>	<u>% of Sampled Students Res- ponding Correctly</u>	<u>Objective</u>	<u>% of Sampled Students Res- ponding Correctly</u>
2-01-1	65	2-14-1	58
2-01-2	83	2-14-2	52
2-01-3	51	2-14-3	84
2-02-1	64	2-14-4	64
2-02-2	62	2-14-5	64
2-02-3	90	2-15-1	81
2-02-4	67	2-15-2	51
2-02-5	76	2-15-3	44
2-02-6	63	2-15-4	65
2-03-1	82	2-15-5	66
2-03-2	64		
2-03-3	91		
2-03-4	86		
2-03-5	81		
2-04-1	40		
2-04-2	41		
2-04-3	62		
2-04-4	57		
2-04-5	60		
2-04-6	73		
2-06-1	81		
2-06-2	74		
2-06-3	56		
2-07-1	46		
2-08-1	27		
2-08-2	21		
2-08-3	42		
2-08-4	86		
2-08-5	79		
2-10-1	63		
2-10-2	41		
2-12-1	37		

APPENDIX E.

ELMR-3

THIRD GRADE

ADMINISTERING INFORMATION

The following instructions to the test administrator assumes the use of machine scorable answer sheets.

The first page of the student booklet introduces him to the answer sheet. Please read this aloud to the class after they have their booklets before them and can follow along. All questions regarding the test format may be answered at this point.

Ball-point or ink will not register on the scoring machine, nor will "X" marks. The child may use a separate sheet for figuring, but should make no random marks on the answer sheet or in the test booklet. The tests are not timed, but the children should be encouraged to work at a steady pace. You may wish to divide the testing session into two parts. This is permissible so long as the booklets and answer sheets are kept secure during the interim.

If the child lacks vocabulary, he may use his text or a dictionary to assist himself, but you should not supply definitions. Frequently, the tests will cover items of learning not a part of your curriculum or in a different format. When this occurs the child should exercise his best logic and intuition to arrive at an answer, but should answer every item on the test.

Items in the test should not be directly discussed before, during or after the test to avoid invalidating this or future tests.

DO NOT OPEN THE TEST BOOKLET UNTIL
YOUR TEACHER TELLS YOU TO

This is an arithmetic test. Read each question or problem carefully. Look at the lettered responses. When you have decided which is the correct answer, blacken in the space between the lines on the answer sheet under the letter of that choice. If you change your mind, erase your first answer completely. Look at the example below.

1. $12 - 5 =$
 A. 12
 B. 5
 C. 7
 D. 0

1. A. B. C. D. E.
 — — — — —

Since 7 is the correct answer, the space under the C in the answer space is blackened.

When your teacher tells you to, open your booklet and begin.

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MATHEMATICS TEST
THIRD GRADE

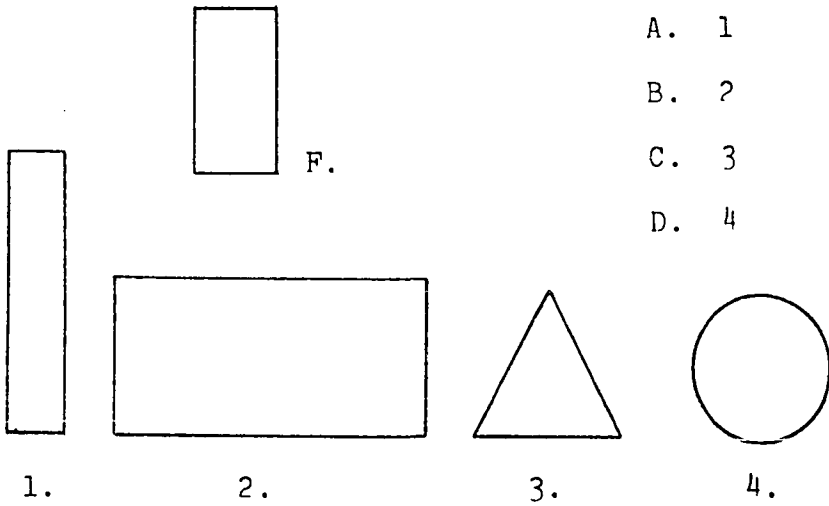
1. Nine thousand forty may be written as
 - A. 9400
 - B. 9004
 - C. 9040
 - D. 904

2. $3 + 9 =$ _____
 - A. 6×2
 - B. $15 - 10$
 - C. $4 + 6$
 - D. $12 - 4$

3. What number times 3 = 3?
 - A. 3
 - B. 2
 - C. 1
 - D. 0

4. Which digit is in the tens place in 1756?
 - A. 1
 - B. 7
 - C. 5
 - D. 6

5. Which figure appears similar to figure F?



6.

$$\square \times 7 = 14$$

The missing numeral is:

- A. 1
- B. 2
- C. 3
- D. 4

7. If we show that 3 pencils cost 25 cents by $\frac{3 \text{ (pencils)}}{25 \text{ cents}}$

how do we show what 9 pencils will cost?

- A. $\frac{50 \text{ cents}}{9 \text{ pencils}}$
- B. $\frac{9 \text{ pencils}}{31 \text{ cents}}$
- C. $\frac{9 \text{ pencils}}{75 \text{ cents}}$
- D. $\frac{31 \text{ cents}}{9 \text{ pencils}}$
8. Find the numeral for six thousand seventy.
- A. 6070
- B. 670
- C. 60070
- D. 6700

9.

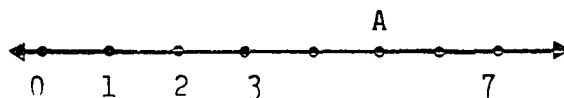
Hundreds	Tens	Ones
111	111	111 11

$$337 = 3 \text{ hundreds} + \underline{\quad} \text{ tens} + \text{ ones.}$$

What should be in the tens place?

- A. 1111
4 tens
- B. 111
3 tens
- C. ~~111~~ ~~111~~ 111
13 tens
- D. ~~111~~ ~~111~~ 11
12 tens

10. 350 is greater than which number?
- A. 345
 - B. 350
 - C. 450
 - D. 360
11. If Ellen had 9 pages of paper dolls with 4 dolls on each page, how many dolls did she have?
- A. 36
 - B. 37
 - C. 18
 - D. 13
12. How many lines can be drawn through the point shown by the dot below?
- A. 1
 - B. 2
 - C. 3
 - D. Many
13. The numeral for point A below can be



- A. 4
- B. 5
- C. 6
- D. 7

14. $500 + 180 + 3 = 600 + \underline{\quad\quad} + 3$. What numeral is missing?

- A. 50
- B. 80
- C. 90
- D. 170

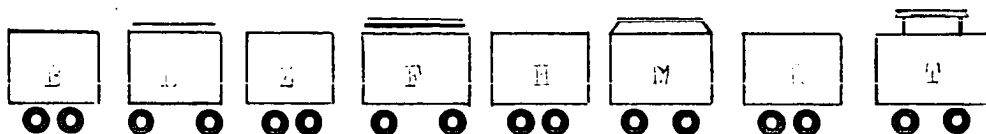
15. The Roman Numeral for seven is

- A. IVI
- B. IIV
- C. VII
- D. IIIIIII

16. $169 < \square$. What number makes the sentence true?

- A. 157
- B. 160
- C. 168
- D. 170

17.



If car B is the fifteenth car in a freight train and car L is the sixteenth, which car is twentieth?

- A. car Z
- B. car T
- C. car H
- D. car M

18. $(4 \times 3) \times 2$ is also equal to:

- A. $4 + (3 + 2)$
- B. $(4 \times 3) \times 3$
- C. $3 + (2 + 4)$
- D. $4 \times (3 \times 2)$

19. Jane can buy 3 pencils for 10¢ at the store. Another way to write "3 for 10¢" is:

- A. 3×10
- B. $\frac{3}{10}$
- C. $10 + 10 + 10$
- D. $\frac{1}{30}$

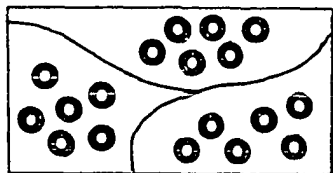
20. Jane bought a doll hat for 39¢. She gave the clerk a dollar. The clerk gave her the hat and what change?

- A. $\textcircled{1\text{¢}}$ $\textcircled{1\text{¢}}$ $\textcircled{1\text{¢}}$ $\textcircled{1\text{¢}}$ $\textcircled{10\text{¢}}$ $\textcircled{25\text{¢}}$
- B. $\textcircled{25\text{¢}}$ $\textcircled{10\text{¢}}$ $\textcircled{10\text{¢}}$ $\textcircled{10\text{¢}}$ $\textcircled{10\text{¢}}$ $\textcircled{1\text{¢}}$
- C. $\textcircled{25\text{¢}}$ $\textcircled{10\text{¢}}$ $\textcircled{1\text{¢}}$
- D. $\textcircled{25\text{¢}}$ $\textcircled{25\text{¢}}$ $\textcircled{10\text{¢}}$ $\textcircled{1\text{¢}}$

21. 3672 can be written as:

- A. $300 + 600 + 70 + 2$
- B. $30 + 60 + 7 + 2$
- C. $3000 + 600 + 7 + 2$
- D. $3000 + 600 + 70 + 2$

22.



This drawing shows:

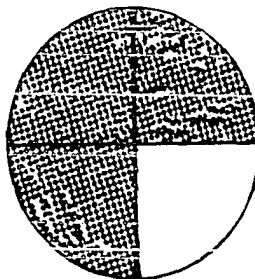
- A. $3 \times 6 = 18$ and $18 \div 6 = 3$
 - B. $2 \times 9 = 18$ and $18 \div 6 = 3$
 - C. $10 \times 8 = 18$ and $18 \div 8 = 10$
 - D. $6 \times 3 = 18$ and $18 \div 9 = 2$
23. How much of the whole picture is shaded?

A. $\frac{3}{5}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{1}{3}$

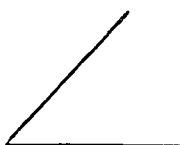


24. What goes in the box to make the statement true?

$$1627 + (240 + 467) = (1627 + \square) + 467$$

- A. 200
- B. 240
- C. 467
- D. 1627

25.

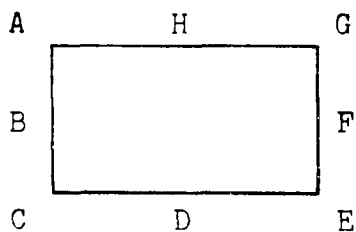


If the angle above can be moved, it will fit on top of:

- A. Angle A
- B. Angle B
- C. Angle C
- D. Angle D

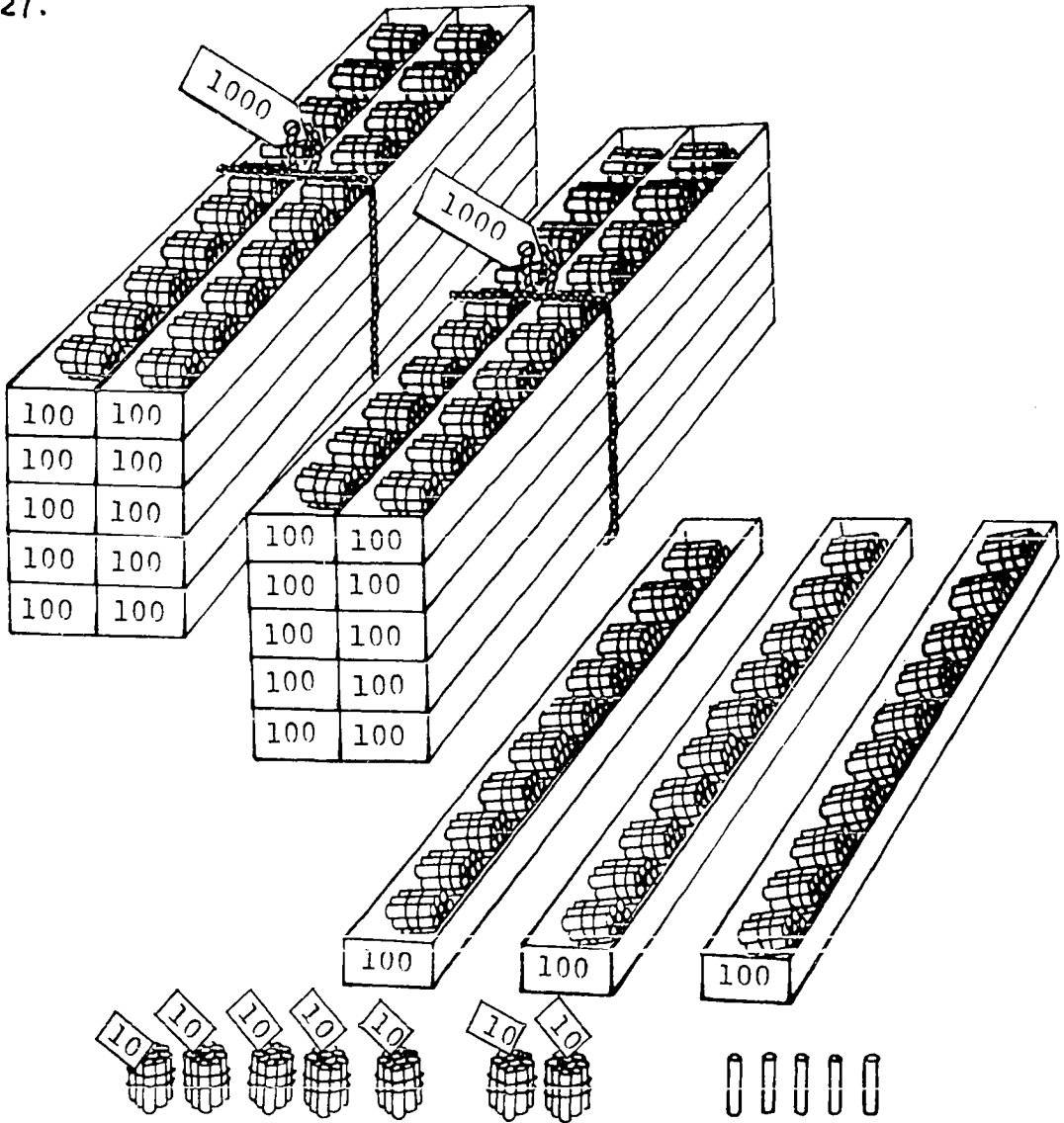


26. The rectangle below can be folded into 2 congruent parts by folding from:



- A. Point B to Point G
- B. Point A to point E.
- C. Point C to point F.
- D. Point H to point E.

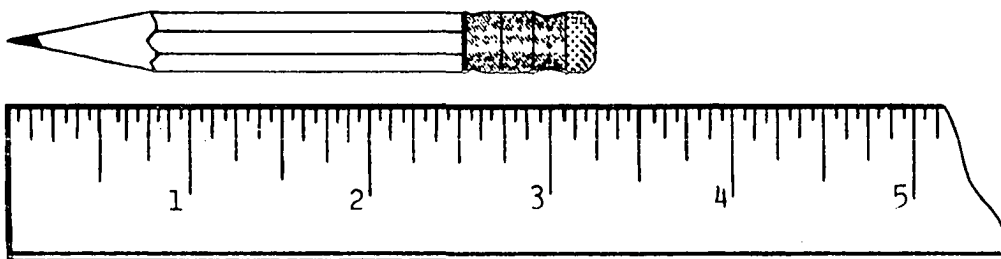
27.



Look at the set of sticks above, what is the total number of sticks in this set?

- A. 2,735
- B. 2,573
- C. 2,375
- D. 2,752

28. Which of the following is the measure of the pencil?



A. $3 \frac{1}{8}$

B. $3 \frac{3}{4}$

C. $3\frac{1}{2}$

D. $3\frac{1}{4}$

29. If $4 \times (5 + 2) = (4 \times 5) + (4 \times n)$, then $n = \underline{\hspace{2cm}}$.

A. 8

B. 5

C. 2

D. 1

30. $37 \times 100 = ?$

A. 37

B. 370

C. 3700

D. 37000

31. $5 \times 4 = n$

A story to fit this sentence would be:

- A. Five boys had 4 pennies each. How many pennies in all did the boys have?
- B. Jack had some pennies. Jim had 5 times as many. How many did Jim have?
- C. Jane had 5 books and Mary had 4. How many books did the two girls have?
- D. Four boys had 20 golf balls each. How many golf balls did they have in all?

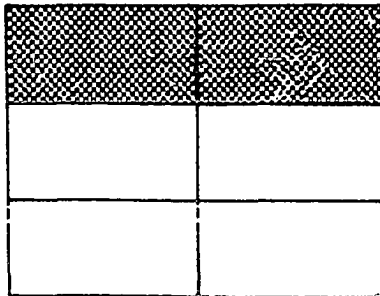
32. Look at the picture at the right. What is shown by this picture?

A. $\frac{2}{3} = \frac{1}{2}$

B. $\frac{2}{6} = \frac{1}{3}$

C. $\frac{4}{6} = \frac{1}{2}$

D. $\frac{2}{6} = \frac{1}{4}$



33. $608 + 297$ is closest to

- A. $600 + 200$
- B. $600 + 300$
- C. $700 + 300$
- D. $950 + 500$

34.

cups	pints	quarts
2	= 1	=
4	= 2	= 1
8	= 4	= 2
12	= 6	= 3

From the information above, figure how many cups would equal 5 pints.

- A. 9
 B. 10
 C. 11
 D. 13
35. What is the missing numeral?

5 thousand, 17 hundreds, 2 tens, 4 ones. =

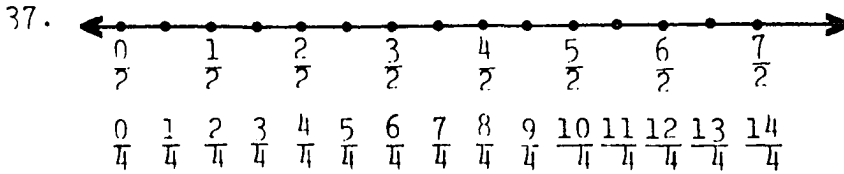
_____ thousands, 7 hundreds, 3 tens, 4 ones.

- A. 6
 B. 5
 C. 7
 D. 4

36. What are the missing numerals in the multiplication table to the right?

X	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	
2	0	2	4	6	8	10		14
3	0	3	6	9	12		18	21
4	0	4	8	12	16	20	24	28
5	0	5	10	15	20	25	30	35
6	0	6	12	18	24	30	36	42
7	0	7	14	21	28	35	42	49

- A. 11, 9, 7
 B. 16, 15, 12
 C. 15, 12, 7
 D. 16, 12, 9



On this number line, the numeral 2 could name the same point as:

- A. $\frac{3}{4}$
- B. $\frac{4}{4}$ or $\frac{2}{2}$
- C. $\frac{3}{2}$ or $\frac{6}{4}$
- D. $\frac{8}{4}$ or $\frac{4}{2}$
38. Which of the following goes into the box to make the statement true?

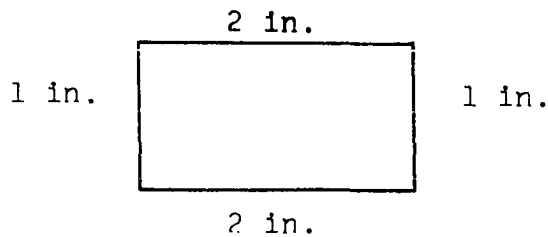
$$36 + 27 \boxed{} 97 - 28$$

A. $<$

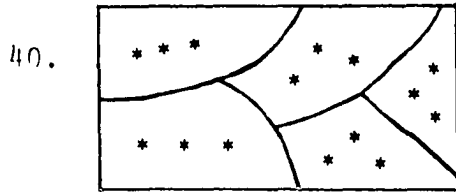
B. $>$

C. $=$

39. The perimeter of the rectangle below is:



- A. 2 square inches
- B. 8 inches
- C. 4 inches
- D. 6 inches



This shows:

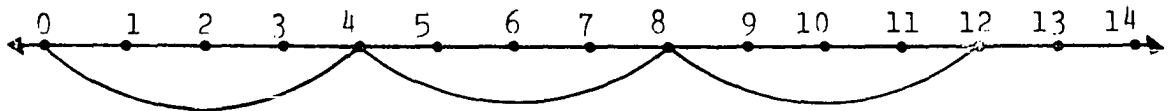
A. $6 \div 3 = n$

B. $15 \div 3 = n$

C. $5 \div 3 = n$

D. $9 \div 3 = n$

41.



This number line picture shows:

A. $3 \times 4 = 12$

B. $4 + 8 = 12$

C. $2 \times 6 = 12$

D. $4 \div 3 = 12$

42. How many straight lines can pass through both of the points X and Y.

A. 1

B. 2

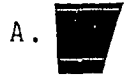
C. None

X

Y

D. Many

43. If the shaded figure were folded to the left over the line XY, which of the following figures would appear?



44. $\frac{1}{2}$ an apple is less than:
- A. $\frac{2}{3}$ of that apple
- B. $\frac{1}{4}$ of that apple
- C. $\frac{1}{6}$ of that apple
- D. $\frac{2}{4}$ of that apple

45. $\begin{array}{r} 379 \\ \times 3 \\ \hline 11\Box7 \end{array}$ What numeral is missing?

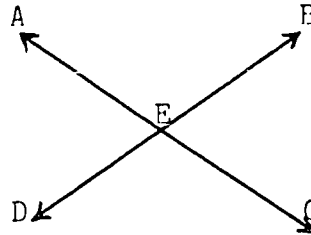
- A. 1
- B. 2
- C. 3
- D. 7

46. To do this exercise, how should the 7 tens and 2 ones be renamed?

$$\begin{array}{r} 7 \text{ tens } 2 \text{ ones} \\ - 4 \text{ tens } 3 \text{ ones} \\ \hline \end{array}$$

- A. 7 tens 12 ones
 B. 6 tens 3 ones
 C. 8 tens 12 ones
 D. 6 tens 12 ones
47. In the figure below, which of the following sets of letters names an angle?

- A. BCD
 B. EBC
 C. CED
 D. CDA



48. Which of the figures below shows a ray?



Figure 1

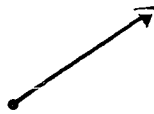


Figure 2

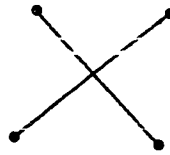


Figure 3



Figure 4

- A. Figure 1
 B. Figure 2
 C. Figure 3
 D. Figure 4

THIRD GRADE
ANSWER KEY

- | | | |
|-------|-------|-------|
| 1. C | 17. D | 33. B |
| 2. A | 18. D | 34. B |
| 3. C | 19. B | 35. A |
| 4. C | 20. D | 36. C |
| 5. B | 21. D | 37. D |
| 6. B | 22. A | 38. A |
| 7. C | 23. C | 39. D |
| 8. A | 24. B | 40. B |
| 9. B | 25. C | 41. A |
| 10. A | 26. B | 42. A |
| 11. A | 27. C | 43. C |
| 12. D | 28. D | 44. A |
| 13. B | 29. C | 45. C |
| 14. B | 30. C | 46. D |
| 15. C | 31. A | 47. C |
| 16. D | 32. B | 48. B |

Following is the coding of items in the 3rd grade test as they relate to the behavioral objectives in Wisconsin K-6 Mathematics Guidelines. The first numeral in the code relates to grade level. The second numeral relates to horizontal strand number (1-15). The strands are found on the fold-out pages. The third numeral is the sub-objective as numbered in the Guideline.

THIRD GRADE

<u>PROBLEM</u>	<u>CODE</u>	<u>PROBLEM</u>	<u>CODE</u>
1.....	3-02-1	25.....	3-10-1
2.....	3-02-2	26.....	3-10-2
3.....	3-04-2	27.....	3-01-1
4.....	3-02-3	28.....	3-14-1
5.....	3-11-1	29.....	3-04-3
6.....	3-15-1	30.....	3-06-4
7.....	3-05-2	31.....	3-15-2
8.....	3-02-1	32.....	3-01-4
9.....	3-15-4	33.....	3-06-5
10.....	3-03-1	34.....	3-14-2
11.....	3-06-1	35.....	3-02-5
12.....	3-08-3	36.....	3-04-4
13.....	3-12-1	37.....	3-12-2
14.....	3-02-5	38.....	3-15-3
15.....	3-02-6	39.....	3-14-3
16.....	3-03-1	40.....	3-04-5
17.....	3-01-2	41.....	3-12-3
18.....	3-04-1	42.....	3-08-2
19.....	3-05-1	43.....	3-09-1
20.....	3-14-4	44.....	3-03-2
21.....	3-02-4	45.....	3-06-3
22.....	3-04-6	46.....	3-06-2
23.....	3-01-3	47.....	3-08-1
24.....	3-15-5	48.....	3-08-1

THIRD GRADE

<u>Objective</u>	<u>% of Sampled Students Res- ponding Correctly</u>	<u>Objective</u>	<u>% of Sampled Students Res- ponding Correctly</u>
3-01-1	63	3-12-1	61
3-01-2	55	3-12-2	17
3-01-3	56	3-12-3	40
3-01-4	36	3-14-1	25
3-02-1	73	3-14-2	40
3-02-2	81	3-14-3	32
3-02-3	76	3-14-4	55
3-02-4	47	3-15-1	87
3-02-5	41	3-15-2	48
3-02-6	46	3-15-3	48
3-03-1	72	3-15-4	75
3-03-2	29	3-15-5	58
3-04-1	63		
3-04-2	65		
3-04-3	35		
3-04-4	53		
3-04-5	40		
3-04-6	53		
3-05-1	47		
3-05-2	62		
3-06-1	61		
3-06-2	24		
3-06-3	29		
3-06-4	52		
3-06-5	39		
3-08-1	31		
3-08-2	38		
3-08-3	64		
3-09-1	28		
3-10-1	64		
3-10-2	55		
3-11-1	28		

APPENDIX F.

ELMR-4

FOURTH GRADE

ADMINISTERING INFORMATION

The following instructions to the test administrator assumes the use of machine scorable answer sheets.

The first page of the student booklet introduces him to the answer sheet. Please read this aloud to the class after they have their booklets before them and can follow along. All questions regarding the test format may be answered at this point.

Ball-point or ink will not register on the scoring machine, nor will "X" marks. The child may use a separate sheet for figuring, but should make no random marks on the answer sheet or in the test booklet. The tests are not timed, but the children should be encouraged to work at a steady pace. You may wish to divide the testing session into two parts. This is permissible so long as the booklets and answer sheets are kept secure during the interim.

If the child lacks vocabulary, he may use his text or a dictionary to assist himself, but you should not supply definitions. Frequently, the tests will cover items of learning not a part of your curriculum or in a different format. When this occurs the child should exercise his best logic and intuition to arrive at an answer, but should answer every item on the test.

Items on the test should not be directly discussed before, during or after the test to avoid invalidating this or future tests.

FOURTH GRADE

DO NOT OPEN THE TEST BOOKLET UNTIL YOUR
TEACHER TELLS YOU TO

Fill in your name and other information requested at the top of the answer sheet.

This is an arithmetic test. Read each question or problem carefully. Look at the lettered responses. When you have decided which is the correct answer, blacken in the space between the lines on the answer sheet under the letter of that choice. If you change your mind, erase your first answer completely. Look at the example below.

1. $12 - 5 =$
 A. 12
 B. 5
 C. 7
 D. 0

1. A. B. **C.** D. E.
 — —  — —

Since 7 is the correct answer, the space under the C in the answer space is blackened.

When your teacher tells you to, open your booklet and begin.

MATHEMATICS TEST
FOURTH GRADE

1. Which of the following expresses 23?

A. $(3 + 4) \times 5$

B. $(3 \times 4) + 5$

C. $3 + (4 \times 5)$

D. $3 \times (4 + 5)$

2. $8 \times 8 = \square$

A. 63

B. 64

C. 72

D. 68

3. $6 \times 9 = \square$

A. 63

B. 64

C. 54

D. 56

4. $8 \times 6 = \square$

A. 45

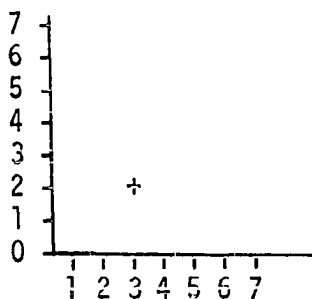
B. 42

C. 56

D. 48

5. Which sentence is true?
- A. 62 seconds is greater than one minute.
 - B. 62 seconds is less than one minute.
 - C. 62 seconds equals one minute.
 - D. None of the above.
6. In the sentence $3 \times \square = 10$, the \square is equal to:
- A. 3
 - B. 4
 - C. 5
 - D. No whole number.
7. $\frac{1}{2}$ of a pie is the same size as:
- A. $\frac{3}{8}$ of that pie
 - B. $\frac{2}{4}$ of that pie
 - C. $\frac{4}{2}$ of that pie
 - D. $\frac{2}{6}$ of that pie
8. In the numeral 76,951,432, the 5 has what place value?
- A. hundreds'
 - B. hundred-thousands'
 - C. thousands'
 - D. ten-thousands'

9. Jim has 4 groups of marbles, of 5 marbles each. He then found 3 more marbles. How many marbles does Jim have in all? Find the number sentence which describes this story.
- A. $(4 + 5) + 3 = \square$
- B. $(4 \times 5) + 3 = \square$
- C. $(5 + 3) \times 4 = \square$
- D. $(3 \times 5) + 4 = \square$
10. If $1367 + 633 = 2000$, then
- A. $2000 - 633 = 1367$
- B. $2000 + 633 = 1367$
- C. $1367 - 2000 = 633$
- D. $2000 + 1367 = 633$
11. Find the solution for \square in the number sentence $(2 \times 4) + \square = 11$.
- A. 5
- B. 2
- C. 3
- D. 4
12. Consider the point in the place below. Which pair of numbers names the point?
- A. (2,4)
- B. (3,3)
- C. (3,2)
- D. (2,2)



13. In the problem $632 > \square$, \square can be:

- A. 622
- B. 652
- C. 642
- D. 632

14. 24 inches =

- A. 2 feet
- B. 1 yard
- C. $\frac{1}{2}$ rod
- D. 3 feet

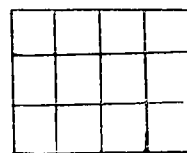
15. Eight pigs are in a pen. They are separated into 4 equal groups.

Choose the number sentence which describes what happened.

- A. $8 \times 4 = r$
- B. $4 \div 8 = r$
- C. $8 \div 4 = r$
- D. $4 \div r = 8$

16. Find the area of this figure:

- A. 14 units
- B. 12 units
- C. 4 units
- D. 3 units



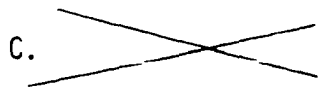
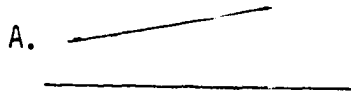
17. One hundred forty-seven thousand twenty-six written as a numeral would be:

- A. 1,470,026
- B. 147,260
- C. 100,047,026
- D. 147,026

18. $6 \times \square = 48$ is equivalent to:

- A. $48 \times 6 = \square$
- B. $48 \div 6 = \square$
- C. $48 \times \square = 6$
- D. $48 - 6 = \square$

19. Find the example of parallel line segments:



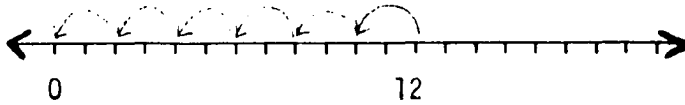
20. What numeral belongs in the box?

- A. 8
- B. 7
- C. 6
- D. 9

$$\begin{array}{r} 64,675 \\ 2,739 \\ + 4,368 \\ \hline \end{array}$$

$$71, \square 82$$

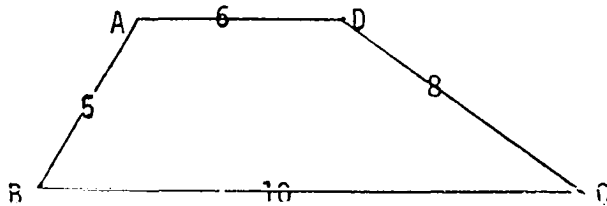
21. This number line illustrates:



- A. 12×2
- B. $12 \div 2$
- C. $12 + 2$
- D. 12×6

22. The perimeter of figure ABCD =

- A. 29
- B. 28
- C. 30
- D. 27



23. Identify the set of whole numbers between 1647 and 1652.

- A. $\{1647, 1648, 1649, 1650, 1651\}$
- B. $\{1647, 1648, 1649, 1650, 1651, 1652\}$
- C. $\{1648, 1649, 1650, 1651, 1652\}$
- D. $\{1648, 1649, 1650, 1651\}$

24. What numeral belongs in the box? $238 \times 100 = \square$

- A. 2,380
- B. 23,800
- C. 238,000
- D. 2,380,000

25. What numeral belongs in the box?

A. 4

B. 5

C. 6

D. 9

$$\begin{array}{r} 5,023 \\ - 2,978 \\ \hline 2,0\boxed{}5 \end{array}$$

26. The number sentence $\frac{3}{4} = \frac{9}{12}$ is true because:

A. $3 \times 4 = 9 \times 12$

B. $3 \times 12 = 4 \times 9$

C. $3 \times 9 = 4 \times 12$

D. $9 \times 3 = 12 \times 4$

27. Identify the set of all factors of 12.

A. ~~1~~, 2, 6, 12~~3~~

B. ~~1~~, 2, 3, 4, 6, 12~~3~~

C. ~~2~~, 3, 4, 6, 12~~3~~

D. ~~1~~, 2, 3, 4, 5, 6, 12~~3~~

28. The Roman numeral that expresses 19 is:

A. IXX

B. XXI

C. XIX

D. XVIII

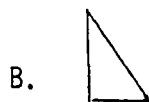
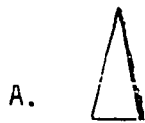
29. Choose the correct numerals for boxes \square and \square in the following problem:

$$\begin{array}{r}
 23 \\
 \times 68 \\
 \hline
 1\square 4 \\
 138 \\
 \hline
 15\square 4
 \end{array}$$

- A. $x = 8$; $y = 5$
 B. $x = 6$; $y = 4$
 C. $x = 8$; $y = 6$
 D. $x = 7$; $y = 5$
30. Find the missing whole number: $\frac{2}{3} = \frac{\square}{9}$

- A. 3
 B. 4
 C. 5
 D. 6

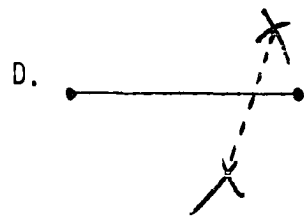
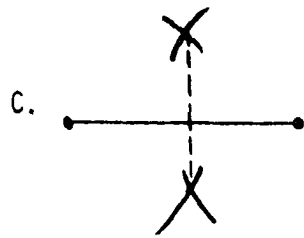
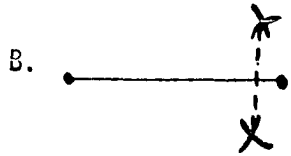
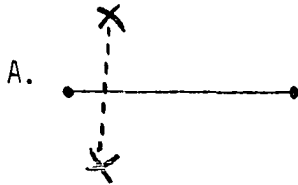
31. Which of the following is an isosceles triangle?



32. Estimate the product of these two numbers: $31 \times 48 = \underline{\hspace{2cm}}$
- A. 1200
 - B. 15,000
 - C. 12,000
 - D. 1500
33. In the sentence $\square \times 23 = 23 \times \square$, the \square is equal to:
- A. 1 only
 - B. 2 only
 - C. 5 only
 - D. Any number
34. In the sentence $5 \times 13 = (\square \times 10) + (\square \times 3)$, the \square is equal to:
- A. 5
 - B. 13
 - C. 3
 - D. 10

35. Construction:

Which of the following demonstrates bisecting a line segment using a compass and straight edge?



36. Identify the set of common factors of 8 and 12.

A. ~~2~~, 4, 8

B. ~~2~~, 3, 4

C. ~~1~~, 2, 4

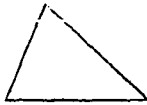
D. ~~1~~, 4, 8

40. Which of the following is a parallelogram?

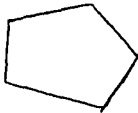
A.



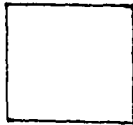
B.



C.



D.



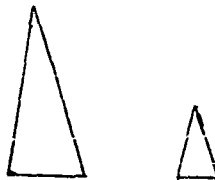
41. These two triangles seem to be:

A. congruent only

B. similar only

C. equal

D. similar & congruent



42. Which of the following can we use to find the number of feet in four yards?

A. $\frac{1}{3} = \frac{4}{\square}$

B. $\frac{1}{3} = \frac{\square}{4}$

C. $\frac{3}{1} = \frac{4}{\square}$

D. $\frac{1}{2} = \frac{4}{\square}$

43. Choose the correct solution to this problem:

$$\begin{array}{r} 32 \overline{) 489} \\ \underline{320} \\ 169 \\ \underline{128} \\ 41 \end{array} \quad \begin{array}{l} 10 \times 32 \\ 5 \times 32 \end{array}$$

$$\begin{array}{r} 32 \overline{) 489} \\ \underline{320} \\ 169 \\ \underline{160} \\ 9 \end{array} \quad \begin{array}{l} 20 \times 32 \\ 5 \times 32 \end{array}$$

$$\begin{array}{r} 32 \overline{) 489} \\ \underline{320} \\ 169 \\ \underline{160} \\ 9 \end{array} \quad \begin{array}{l} 10 \times 32 \\ 5 \times 32 \end{array}$$

$$\begin{array}{r} 32 \overline{) 489} \\ \underline{320} \\ 169 \\ \underline{160} \\ 9 \end{array} \quad \begin{array}{l} 100 \times 32 \\ 50 \times 32 \end{array}$$

44. A sheet of paper is a model of:

- A. a point
- B. a line
- C. a plane
- D. space

45. Estimate the answer for: $798 \div 22 = \underline{\hspace{2cm}}$

- A. 44
- B. 40
- C. 400
- D. 30

46. $n \div 1 = \square$. When n does not equal 0 or 1, \square is equal to:

- A. 1
- B. n
- C. 0
- D. $\frac{1}{n}$

47. Which figure or figures have two or more axes of symmetry?

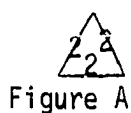


Figure A

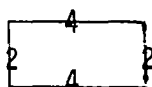


Figure B



Figure C

- A. A and B
 - B. A and C
 - C. B and C
 - D. A, B, and C
48. The set of points where two planes intersect represents:
- A. a point
 - B. a line
 - C. a plane
 - D. space
49. The set of points in a plane that are all the same distance from one point is called:
- A. a square
 - B. a line
 - C. a triangle
 - D. a circle

37. In the sentence, $\underline{\quad} \times \underline{\quad} = 30$,
- A. All of the answers given are correct.
- B. $\underline{\quad} = 3$, $\underline{\quad} = 10$
- C. $\underline{\quad} = 5$, $\underline{\quad} = 6$
- D. $\underline{\quad} = 10$, $\underline{\quad} = 3$
38. Choose the problem that is best represented by the following number sentence: $\underline{\quad} \times 3 = 12$.
- A. Jane had some jacks. Mary gave her three more jacks. She now has 12 jacks. How many jacks did Jane have to begin with?
- B. There are 12 marbles in all. There are 3 marbles in each group. How many groups of marbles are there?
- C. Bill sold 3 times as many tickets as Bob. Bob sold 12 tickets. How many tickets did Bill sell?
- D. The children bought 12 different things at the dime store. The clerk put 3 things into one bag. If she put the rest into a different bag, how many did she put in this bag?

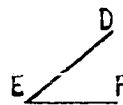
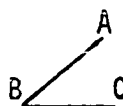
39. These angles seem to be:

A. perpendicular

B. congruent

C. obtuse

D. vertical



50. Look at the line segment \overline{AB} . Which statement is true?



- A. There is one point between point A and point B
 - B. There are 3 points between point A and point B
 - C. There are no points between point A and point B.
 - D. There are many points between point A and point B.
51. The set of all points is:
- A. a point
 - B. a line
 - C. a plane
 - D. space

FOURTH GRADE
ANSWER KEY

1. C	18. B	35. C
2. B	19. B	36. C
3. C	20. B	37. A
4. D	21. B	38. B
5. A	22. A	39. B
6. D	23. D	40. D
7. B	24. B	41. B
8. D	25. A	42. A
9. B	26. B	43. C
10. A	27. B	44. C
11. C	28. C	45. B
12. C	29. C	46. B
13. A	30. D	47. D
14. A	31. A	48. B
15. C	32. D	49. D
16. B	33. D	50. D
17. D	34. A	51. D

Following is the coding of items in the 4th grade test as they relate to the behavioral objectives in Wisconsin K-6 Mathematics Guidelines. The first numeral in the code relates to grade level. The second numeral relates to horizontal strand number (1-15). The strands are found on the fold-out pages. The third numeral is the sub-objective as numbered in the Guideline.

FOURTH GRADE

<u>PROBLEM</u>	<u>CODE</u>	<u>PROBLEM</u>	<u>CODE</u>
1.....	4-04-3	26.....	4-05-2
2.....	4-06-1	27.....	4-01-1
3.....	4-06-1	28.....	4-02-3
4.....	4-06-1	29.....	4-06-5
5.....	4-14-1	30.....	4-05-3
6.....	4-15-7	31.....	4-07-1
7.....	4-05-1	32.....	4-06-7
8.....	4-02-2	33.....	4-15-4
9.....	4-15-9	34.....	4-15-5
10.....	4-04-1	35.....	4-13-3
11.....	4-15-8	36.....	4-01-2
12.....	4-12-2	37.....	4-15-6
13.....	4-03-1	38.....	4-15-2
14.....	4-14-2	39.....	4-10-1
15.....	4-15-2	40.....	4-07-1
16.....	4-14-4	41.....	4-11-1
17.....	4-02-1	42.....	4-05-4
18.....	4-15-3	43.....	4-06-6
19.....	4-08-5	44.....	4-08-2
20.....	4-06-2	45.....	4-06-7
21.....	4-12-3	46.....	4-04-2
22.....	4-14-3	47.....	4-09-1
23.....	4-03-1	48.....	4-08-3
24.....	4-06-4	49.....	4-08-4
25.....	4-06-3	50.....	4-12-1
		51.....	4-08-1

FOURTH GRADE

<u>Objective</u>	<u>% of Sampled Students Res- ponding Correctly</u>	<u>Objective</u>	<u>% of Sampled Students Res- ponding Correctly</u>
4-01-1	38	4-13-3	60
4-01-2	25	4-14-1	78
4-02-1	47	4-14-2	71
4-02-2	41	4-14-3	68
4-02-3	42	4-14-4	81
4-03-1	44	4-15-2	49
4-04-1	73	4-15-3	66
4-04-2	25	4-15-4	41
4-04-3	88	4-15-5	32
4-05-1	70	4-15-6	33
4-05-2	33	4-15-7	82
4-05-3	41	4-15-8	86
4-05-4	20	4-15-9	66
4-06-1	89		
4-06-2	53		
4-06-3	41		
4-06-4	42		
4-06-5	45		
4-06-6	28		
4-06-7	31		
4-07-1	39		
4-08-1	22		
4-08-2	28		
4-08-3	26		
4-08-4	11		
4-08-5	54		
4-09-1	26		
4-10-1	25		
4-11-1	30		
4-12-1	19		
4-12-2	55		
4-12-3	43		

APPENDIX G.

ELMR-5

ADMINISTERING INFORMATION

The following instructions to the test administrator assumes the use of machine scorable answer sheets.

The first page of the student booklet introduces him to the answer sheet. Please read this aloud to the class after they have their booklets before them and can follow along. All questions regarding the test format may be answered at this point.

Ball-point or ink will not register on the scoring machine, nor will "X" marks. The child may use a separate sheet for figuring, but should make no random marks on the answer sheet or in the test booklet. The tests are not timed, but the children should be encouraged to work at a steady pace. You may wish to divide the testing session into two parts. This is permissible so long as the booklets and answer sheets are kept secure during the interim.

If the child lacks vocabulary, he may use his text or a dictionary to assist himself, but you should not supply definitions. Frequently, the tests will cover items of learning not a part of your curriculum or in a different format. When this occurs the child should exercise his best logic and intuition to arrive at an answer, but should answer every item on the test.

Items in the test should not be directly discussed before, during or after the test to avoid invalidating this or future tests.

FIFTH GRADE TEST

DO NOT OPEN THE TEST BOOKLET
UNTIL YOUR TEACHER TELLS YOU TO

Fill in your name and other information requested at the top of the answer sheet.

This is an arithmetic test. Read each question or problem carefully. Look at the lettered responses. When you have decided which is the correct answer, blacken in the space between the lines on the answer sheet under the letter of that choice. If you change your mind, erase your first answer completely. Look at the example below.

1. $12 - 5 =$

A. 12

B. 5

C. 7

D. 0

i. A. B. C. D. E.

Since 7 is the correct answer, the space under the C in the answer space is blackened.

When your teacher tells you to, open your booklet and begin.

MATHEMATICS TEST
FIFTH GRADE

1. $24 \div 5 = \underline{\quad ? \quad}$

336

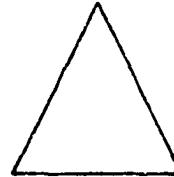
A. $4 \frac{2}{5}$

B. $4 \frac{4}{5}$

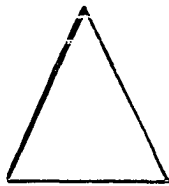
C. $5 \frac{2}{5}$

D. $5 \frac{4}{5}$

2. The triangle at the right appears to be congruent with which triangle below:



A.



B.



C.



D.

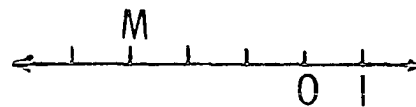
3. On the number line to the right, point M represents:

A. -4

B. -3

C. -1

D. 3



4. Let $\frac{A}{B}$ be any fraction, then $\frac{A}{B} + 0 = \underline{\quad ? \quad}$

A. $\frac{B}{A}$

B. $\frac{A}{B}$

C. 0

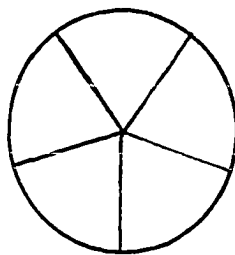
5. The picture at the right represents a pie. If three people are each served one piece from the pie, which sentence illustrates the situation?

A. $1 \div 3 = \frac{1}{3}$

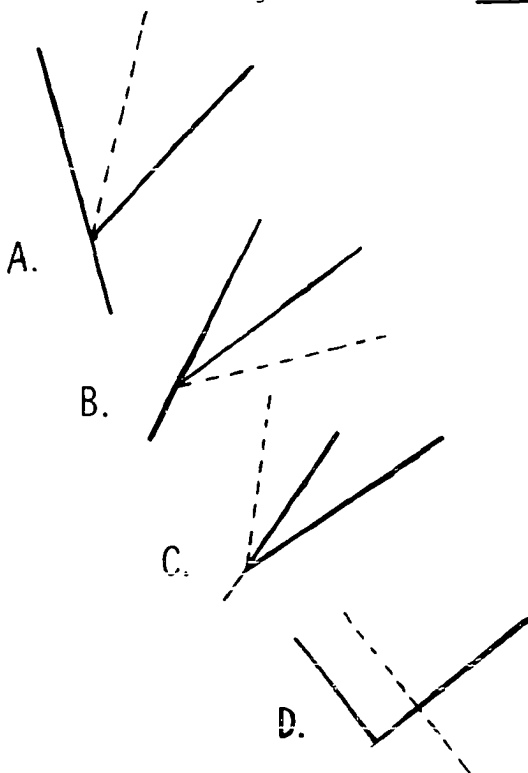
B. $\frac{5}{5} = \frac{3}{5} = \frac{2}{5}$

C. $\frac{5}{5} = \frac{2}{5} = \frac{3}{5}$

D. $\frac{2}{5} + \frac{3}{5} = \frac{5}{5}$



6. Which of the following demonstrates bisecting an angle?



7. Which one of the following fractions belong to this set of equivalent fractions? $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{5}{10}$

A. $\frac{4}{6}$

B. $\frac{4}{8}$

C. $\frac{4}{12}$

D. $\frac{4}{16}$

8. How do the fractions $\frac{3}{8}$ and $\frac{3}{4}$ compare?

A. $\frac{3}{8} = \frac{3}{4}$

B. $\frac{3}{8} < \frac{3}{4}$

C. $\frac{3}{8} > \frac{3}{4}$

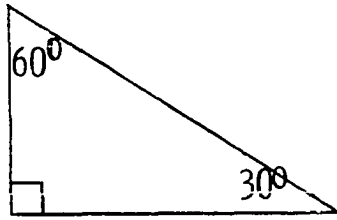
9. A right triangle is pictured below; a right angle contains:

A. 30°

B. 60°

C. 90°

D. 180°



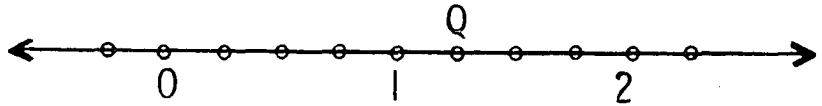
10. Point Q represents what number?

A. $\frac{3}{4}$

B. $\frac{4}{4}$

C. $\frac{5}{4}$

D. $\frac{7}{4}$



11. The instrument which permits exact measurements is:

A. Ruler.

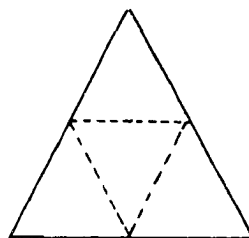
B. Cloth tape.

C. Metal tape, because it does not stretch.

D. None of the above, because no measuring instrument is exact.

12. If the figure at the right were folded on the dashed lines and the edges taped, it would form a:

- A. Dodecahedron
 B. Cube
 C. Pyramid
 D. Trapezoid



13. Which is true?

- A. $\frac{4}{2} + \frac{1}{4} = \frac{5}{6}$
 B. $\frac{2}{4} + \frac{1}{4} = \frac{1}{4} + \frac{2}{4}$
 C. $\frac{1}{4} + \frac{4}{2} = \frac{3}{4}$
 D. $\frac{4}{2} + \frac{4}{1} = \frac{8}{2}$

14. Which of the following lists of numbers contain only prime numbers?

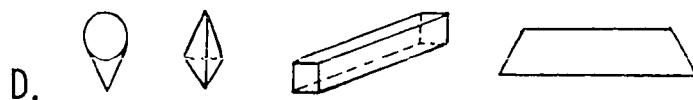
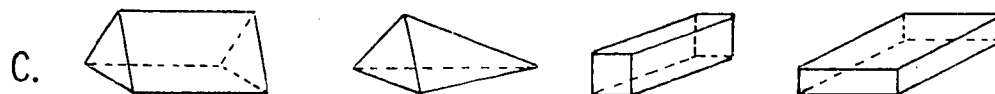
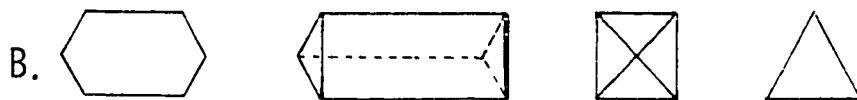
- A. 3, 5, 7, 9
 B. 13, 17, 19, 23
 C. 11, 13, 15, 19
 D. 10, 20, 30, 40

15. Select the correct numeral for boxes \boxed{x} and \boxed{y} in the following problem.

- A. $x = 2, y = 4$
 B. $x = 1, y = 3$
 C. $x = 2, y = 3$
 D. $x = 3, y = 4$

$$\begin{array}{r}
 4289 \\
 \times 375 \\
 \hline
 21445 \\
 300\boxed{x}3 \\
 \hline
 12867 \\
 \hline
 1608\boxed{y}75
 \end{array}$$

16. In which group are all four figures polyhedrons?



17. The measure of \overline{AB} is best approximated as:

- A. 2 units
- B. 3 units
- C. 4 units
- D. 5 units



18. Select the numeral which is not a name for $\frac{3}{4}$.

- A. $\frac{1}{4} + \frac{2}{4}$
- B. $1 - \frac{1}{4}$
- C. $\frac{9}{12}$
- D. $\frac{18}{22}$

19. Which is true?

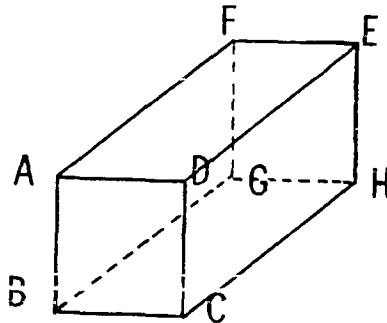
- A. $\frac{1}{2} + (\frac{1}{4} + \frac{1}{3}) = (\frac{1}{2} - \frac{1}{4}) + \frac{1}{3}$
 B. $(\frac{1}{3} + \frac{1}{2}) + \frac{1}{4} = (\frac{1}{4} - \frac{1}{3}) + \frac{1}{2}$
 C. $(\frac{1}{2} + \frac{1}{3}) + \frac{1}{4} = \frac{1}{2} + (\frac{1}{3} + \frac{1}{4})$
 D. $(\frac{1}{2} + \frac{1}{3}) + (\frac{1}{3} + \frac{1}{4}) = \frac{1}{2} (\frac{1}{3} + \frac{1}{4})$

20. Select the least common multiple of 15 and 25:

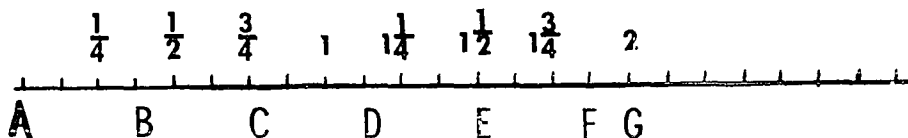
- A. 45
 B. 75
 C. 125
 D. 375

21. In the figure shown \overline{CH} is a (an)

- A. edge
 B. face
 C. vertex
 D. diagonal



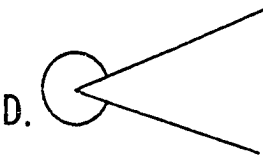
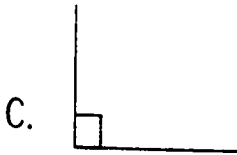
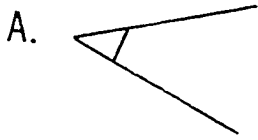
22. Given the following number line:



Point G on the above line represents:

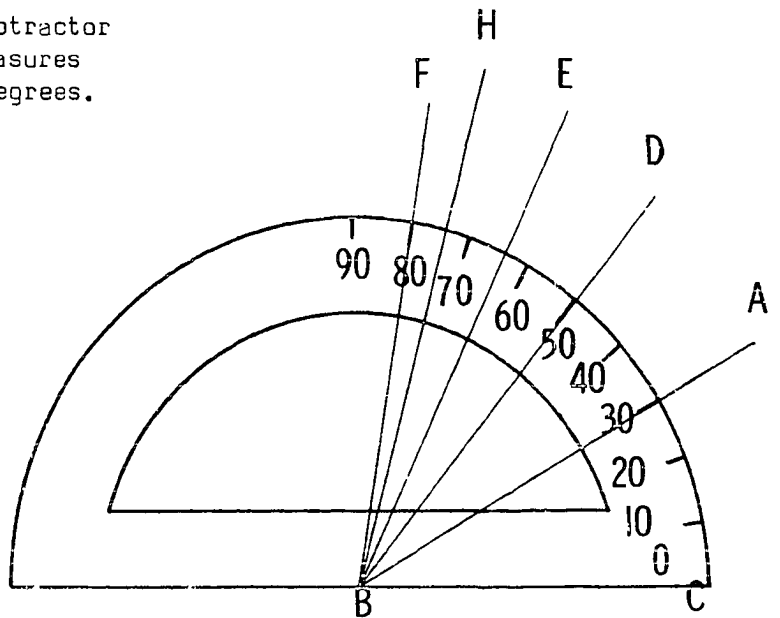
- A. $\frac{1}{8}$
 B. $\frac{2}{8}$
 C. $\frac{3}{8}$
 D. $\frac{8}{4}$

23. Which of the following is an example of a right angle?



24. In the figure of the protractor to the right, $\angle CBE$ measures approximately _____ degrees.

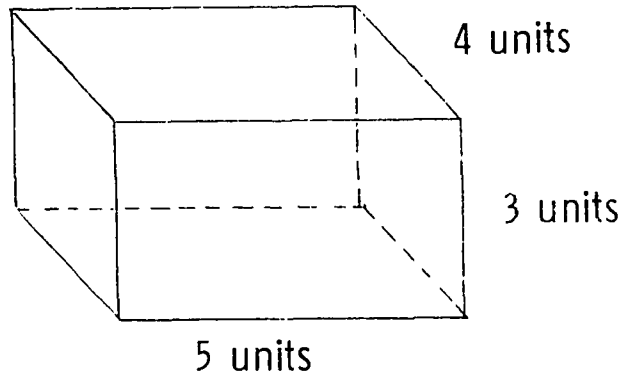
- A. 20
- B. 35
- C. 65
- D. 75



25. What is the greatest common factor of 15 and 75?
- A. 25
 - B. 15
 - C. 5
 - D. 3
26. Which of the subtraction problems below cannot be done in the set of positive rational numbers?
- A. $\frac{3}{4} - \frac{1}{8} =$
 - B. $\frac{1}{2} - \frac{3}{8} =$
 - C. $\frac{4}{5} - \frac{7}{8} =$
 - D. $\frac{5}{9} - \frac{1}{9} =$
27. $\frac{3}{4}$ may be expressed as:
- A. $\frac{9}{12}$ and $\frac{15}{20}$
 - B. $\frac{6}{12}$ and $\frac{12}{24}$
 - C. $\frac{9}{16}$ and $\frac{27}{48}$
 - D. $\frac{6}{16}$ and $\frac{18}{48}$

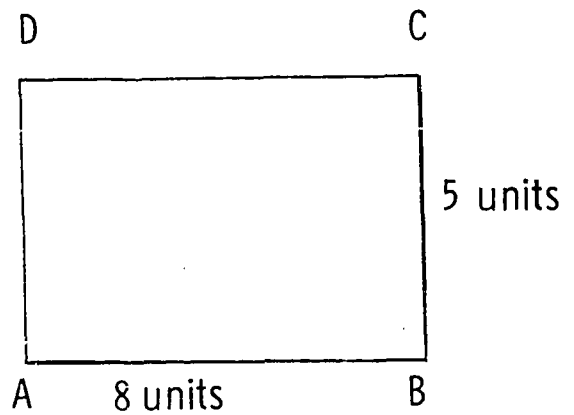
28. The volume of the rectangular solid is:

- A. 80 cubic units
- B. 60 cubic units
- C. 24 cubic units
- D. 12 cubic units



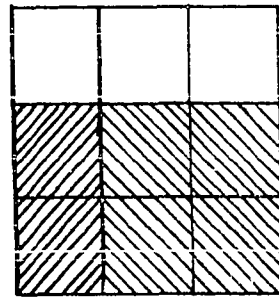
29. The area of rectangle ABCD is:

- A. 13 square units
- B. 20 square units
- C. 26 square units
- D. 40 square units



30. The number 1 is assigned to this region.
Study the shaded portion and select the correct equation.

- A. $\frac{2}{9} + \frac{4}{9} = \frac{6}{9}$
- B. $\frac{2}{9} + \frac{4}{9} = \frac{6}{18}$
- C. $\frac{3}{9} + \frac{4}{9} = \frac{7}{9}$
- D. $\frac{3}{9} + \frac{4}{9} = \frac{6}{9}$



31. Choose the correct solution to this problem: 24 $\overline{4862}$

A. 24 $\overline{4862}$
 $\begin{array}{r} 4800 \\ \underline{62} \\ 72 \end{array}$ $\begin{array}{l} 200 \times 24 \\ 3 \times 24 \\ \hline 203 \end{array}$

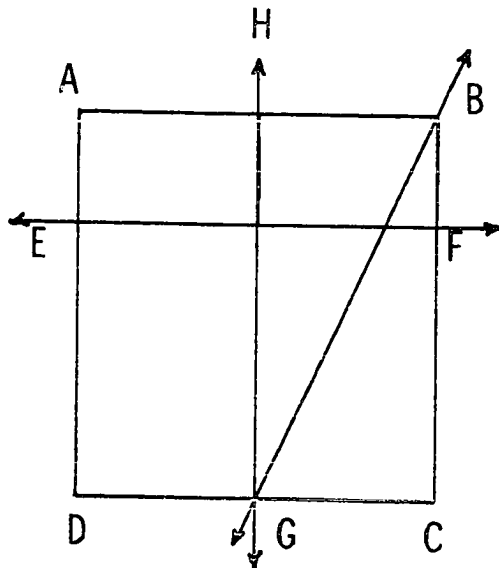
B. 24 $\overline{4862}$
 $\begin{array}{r} 7200 \\ \underline{662} \\ 480 \\ \underline{182} \\ 168 \\ \underline{14} \end{array}$ $\begin{array}{l} 300 \times 24 \\ 20 \times 24 \\ 7 \times 24 \\ \hline 327 \end{array}$

C. 24 $\overline{4862}$
 $\begin{array}{r} 4800 \\ \underline{62} \\ 48 \\ \underline{14} \end{array}$ $\begin{array}{l} 200 \times 24 \\ 2 \times 24 \\ \hline 202 \end{array}$

D. 24 $\overline{4862}$
 $\begin{array}{r} 2400 \\ \underline{2462} \\ 4800 \\ \underline{62} \\ 48 \\ \underline{14} \end{array}$ $\begin{array}{l} 100 \times 24 \\ 200 \times 24 \\ 2 \times 24 \\ \hline 302 \end{array}$

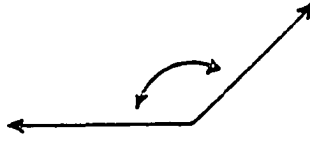
32. Locate rectangle A B C D in the figure below. Which of the following lines separates the rectangle into symmetric figures?

- A. A B
- B. B G
- C. E F
- D. G H



33. What type of angle is this?

- A. Obtuse
- B. Acute
- C. Right
- D. Straight



34. The smallest positive rational number is:

- A. any positive number less than $\frac{1}{2}$
- B. $\frac{1}{9999999}$
- C. 0
- D. impossible to find

35. Which of the following shows that $\frac{3}{4} > \frac{2}{5}$.

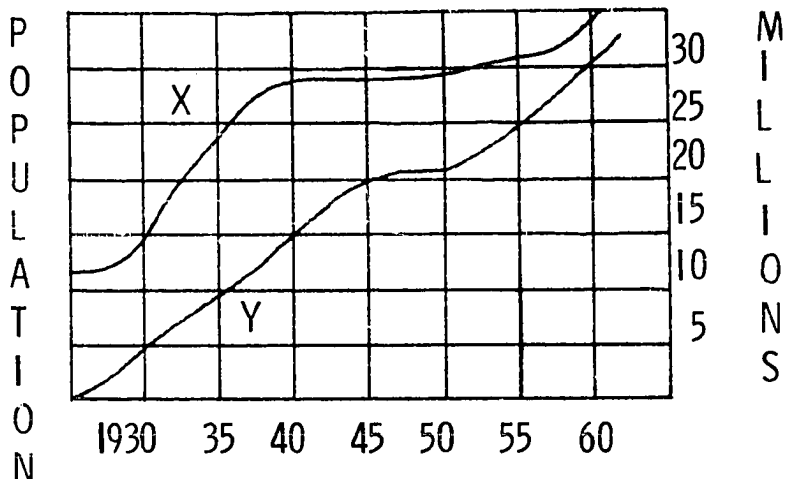
- A. $\frac{6}{4} > \frac{4}{5}$
- B. $\frac{15}{20} > \frac{8}{20}$
- C. $\frac{6}{8} > \frac{6}{10}$
- D. $\frac{3}{2} > \frac{2}{4}$

36. Select the number which is not a composite number.

- A. 19
- B. 21
- C. 27
- D. 36

37. During which 5 year period did country X witness its greatest population growth?

- A. 1935-1940
- B. 1940-1945
- C. 1945-1950
- D. 1950-1955



38. (The graph above is used for questions 37 and 38.)

The least difference in population between the two countries occurred in:

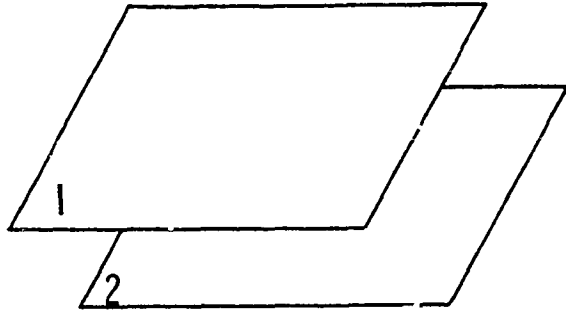
- A. 1930
- B. 1940
- C. 1950
- D. 1960

39. Using the formula, $E = V + F - 2$, find the number of edges (E) in a polyhedron that has 8 vertices, (V), and 6 faces, (F).

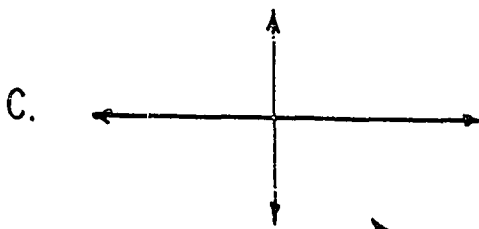
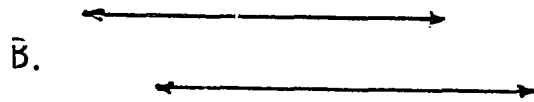
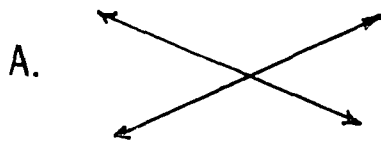
- A. 16
- B. 14
- C. 12
- D. 10

40. In the portions at the right, plane 1 and plane 2 appear to:

- A. be parallel
- B. be perpendicular
- C. intersect



41. Look at the pairs of lines below. Which pair appears to be perpendicular?



42. The sentence $(2 \times \frac{1}{3}) + (2 \times \frac{2}{3}) = 2$ is correct. Which of the following shows the proper use of the distributive property in simplifying the sentence?

A. $(2 + 2) + (\frac{1}{3} + \frac{2}{3}) = 2$

B. $2 \times (\frac{1}{3} + \frac{2}{3}) = 2$

C. $(\frac{1}{3} \times 2) + \frac{2}{3} \times 2 = 2$

D. $2 + (\frac{1}{3} + 2) \times \frac{2}{3} = 2$

43. The sentence $\frac{1}{2} \times \square = \frac{3}{4}$ is equivalent to:

A. $\frac{3}{4} \times \frac{1}{2} = \square$

B. $\frac{1}{2} \div \frac{3}{4} = \square$

C. $\frac{1}{2} \times \frac{3}{4} = \square$

D. $\frac{3}{4} \div \frac{1}{2} = \square$

44. If $N \div \frac{3}{4} = 1$, then N equals:

A. $\frac{1}{4}$

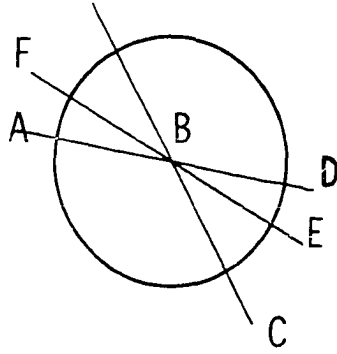
B. $\frac{2}{4}$

C. $\frac{3}{4}$

D. $\frac{4}{4}$

45. In the circle below, which line segment is a radius?

- A. BD
- B. AC
- C. CD
- D. EF

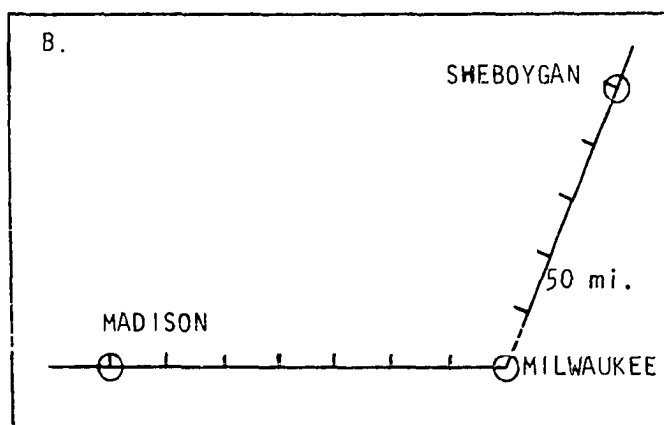
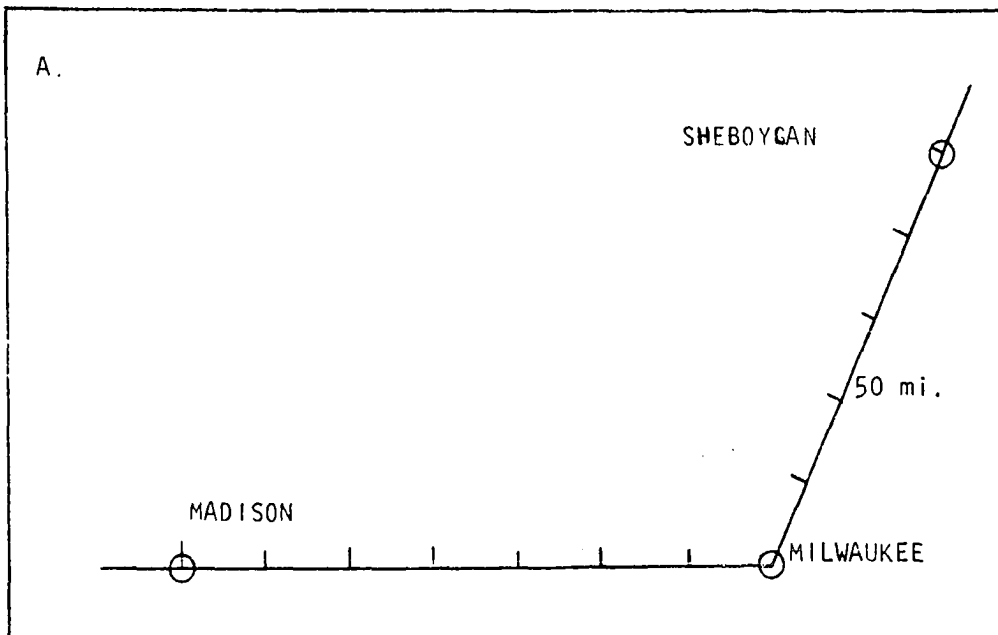


46. The Roman numeral that expresses the number 66 is:

- A. LXVI
- B. LXIV
- C. XLIV
- D. XLVI

47. Which of the subtraction exercises below is correct?

- A. $\frac{5}{6} - \frac{2}{3} = \frac{3}{6}$
- B. $\frac{5}{8} - \frac{1}{2} = \frac{4}{6}$
- C. $\frac{7}{8} - \frac{1}{4} = \frac{5}{8}$
- D. $\frac{4}{5} - \frac{7}{10} = \frac{1}{20}$



48. Which map shows that the distance from Madison to Milwaukee is approximately 70 miles?

- A. A only
- B. B only
- C. Neither
- D. Both

49. A plane is determined by _____ point(s) not in a straight line.
- A. Zero
 - B. One
 - C. Two
 - D. Three
50. Select the numeral which correctly expresses eighty-nine and forty-seven thousandths.
- A. 89.470
 - B. 89.47
 - C. 89.047
 - D. 89.0047
51. 2^3 is another name for:
- A. 2000
 - B. 9
 - C. 8
 - D. 6
52. The prime factors of 45 are:
- A. 1, 5, 45
 - B. 5, 9
 - C. 3, 5
 - D. 1, 3, 15

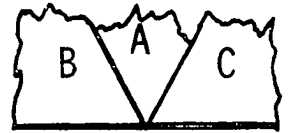
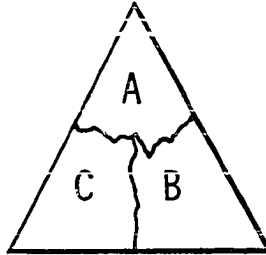
53. In the sentence $\frac{3}{4} = \frac{\square}{35}$, the \square represents:

- A. $\frac{5}{4}$
- B. $\frac{15}{4}$
- C. $\frac{21}{4}$
- D. $\frac{27}{4}$

54. Which of the following is true for the sentence $\frac{5}{8} = \frac{3}{\square}$?

- A. $5 \times \square = 3 \times 8$
- B. $\square \times 8 = 5 \times 3$
- C. $3 \times \square = 5 \times 8$

55. If the triangle below is torn apart on the crooked lines and put together again, like the figure at the right, what is the sum of the measures of the angles?



- A. 90°
- B. 180°
- C. 360°
- D. it varies

56. The numeral 32 in base five, (32_{five}) is equivalent to what base ten numeral?

- A. 5
- B. 12
- C. 17
- D. 32

FIFTH GRADE
ANSWER KEY

1. B	21. A	41. C
2. B	22. D	42. B
3. B	23. C	43. D
4. B	24. C	44. C
5. B	25. B	45. A
6. A	26. C	46. A
7. B	27. A	47. C
8. B	28. B	48. D
9. C	29. D	49. D
10. C	30. A	50. C
11. D	31. C	51. C
12. C	32. D	52. C
13. B	33. A	53. B
14. B	34. D	54. A
15. C	35. B	55. B
16. C	36. A	56. C
17. B	37. A	
18. D	38. D	
19. C	39. C	
20. B	40. A	

Following is the coding of items in the fifth grade test as they relate to the behavioral objectives in Wisconsin K-6 Mathematics Guidelines. The first numeral in the code relates to grade level. The second numeral relates to horizontal strand number (1-15). The strands are found on the fold-out pages. The third numeral is the sub-objective as numbered in the Guideline.

FIFTH GRADE

<u>PROBLEM</u>	<u>CODE</u>	<u>PROBLEM</u>	<u>CODE</u>
1.....	5-06-4	29.....	5-14-3
2.....	5-10-1	30.....	5-04-6
3.....	5-12-3	31.....	5-06-3
4.....	5-04-2	32.....	5-09-1
5.....	5-15-2	33.....	5-08-1
6.....	5-13-2	34.....	5-04-5
7.....	5-01-6	35.....	5-05-3
8.....	5-03-1	36.....	5-01-3
9.....	5-14-5	37.....	5-12-1
10.....	5-04-1	38.....	5-12-1
11.....	5-14-7	39.....	5-07-3
12.....	5-13-1	40.....	5-08-2
13.....	5-04-4	41.....	5-08-3
14.....	5-01-1	42.....	5-15-3
15.....	5-06-2	43.....	5-15-1
16.....	5-07-1	44.....	5-15-1
17.....	5-14-6	45.....	5-08-4
18.....	5-02-1	46.....	5-02-5
19.....	5-04-4	47.....	5-06-1
20.....	5-01-4	48.....	5-11-1
21.....	5-07-2	49.....	5-08-5
22.....	5-12-2	50.....	5-02-4
23.....	5-08-1	51.....	5-02-3
24.....	5-14-1	52.....	5-01-2
25.....	5-01-5	53.....	5-05-1
26.....	5-04-3	54.....	5-05-2
27.....	5-06-5	55.....	5-14-2
28.....	5-14-4	56.....	5-02-2

FIFTH GRADE

<u>Objective</u>	<u>% of Sampled Students Res- ponding Correctly</u>	<u>Objective</u>	<u>% of Sampled Students Res- ponding Correctly</u>
5-01-1	43	5-08-1	51
5-01-2	21	5-08-2	37
5-01-3	51	5-08-3	30
5-01-4	41	5-08-4	35
5-01-5	37	5-08-5	25
5-01-6	69	5-09-1	36
5-02-1	49	5-10-1	92
5-02-2	15	5-11-1	42
5-02-3	16	5-12-1	39
5-02-4	23	5-12-2	37
5-02-5	46	5-12-3	42
5-03-1	48	5-13-1	48
5-04-1	36	5-13-2	42
5-04-2	79	5-14-1	58
5-04-3	39	5-14-2	12
5-04-4	41	5-14-3	29
5-04-5	38	5-14-4	20
5-04-6	46	5-14-5	53
5-05-1	31	5-14-6	40
5-05-2	20	5-14-7	51
5-05-3	38	5-15-1	33
5-06-1	27	5-15-2	61
5-06-2	55	5-15-3	34
5-06-3	49		
5-06-4	68		
5-06-5	50		
5-07-1	54		
5-07-2	49		
5-07-3	32		

APPENDIX H.

ELMR-6

SIXTH GRADE

ADMINISTERING INFORMATION

The following instructions to the test administrator assumes the use of machine scorable answer sheets.

The first page of the student booklet introduces him to the answer sheet. Please read this aloud to the class after they have their booklets before them and can follow along. All questions regarding the test format may be answered at this point.

Ball-point or ink will not register on the scoring machine, nor will "X"marks. The child may use a separate sheet for figuring, but should make no random marks on the answer sheet or in the test booklet. The tests are not timed, but the children should be encouraged to work at a steady pace. You may wish to divide the testing session into two parts. This is permissible so long as the booklets and answer sheets are kept secure during the interim.

If the child lacks vocabulary, he may use his text or a dictionary to assist himself, but you should not supply definitions. Frequently, the tests will cover items of learning not a part of your curriculum or in a different format. When this occurs the child should exercise his best logic and intuition to arrive at an answer, but should answer every item on the test.

Items in the test should not be directly discussed before, during or after the test to avoid invalidating this or future tests.

SIXTH GRADE TEST

DO NOT OPEN THE TEST BOOKLET UNTIL
YOUR TEACHER TELLS YOU TO

Fill in your name and other information
requested at the top of the answer sheet.

This is an arithmetic test. Read each
question or problem carefully. Look at the
lettered responses. When you have decided
which is the correct answer, blacken in
the space between the lines on the answer
sheet under the letter of that choice. If
you change your mind, erase your first
answer completely. Look at the example
below.

1. $12 - 5 =$

- A. 12
- B. 5
- C. 7
- D. 0

1. A. B. C. D. E.

Since 7 is the correct answer, the space
under the C in the answer space is blackened.

When your teacher tells you to, open your
booklet and begin.

MATHEMATICS TEST
SIXTH GRADE

361

1. If you subtract 5 from 3, the answer should be:
- A. An exponent
 - B. A negative number
 - C. Zero
 - D. A positive number
2. Three-fifths is represented by:
- A. .6
 - B. .3
 - C. .06
 - D. .03
3. The speed of light is given as $186000 \frac{\text{miles}}{\text{second}}$.
Written in scientific notation this would appear as:
- A. 1.86×10^5
 - B. 1.86×10^4
 - C. 1.86×10^3
 - D. 1.86×10^2
4. 3564 expressed in exponential notation would be:
- A. $(3 \times 10^3) + (5 \times 10^2) + (6 \times 10) + 4$
 - B. $(3 \times 10^2) + (5 \times 10^1) + (6 \times 1) + 4$
 - C. $(3 \times 10^1) + (5 \times 10^0) + (6 \times 1) + 4$
 - D. $(3 \times 10^0) + (5 \times 10^{-1}) + (6 \times 1) + 4$

5. The common fraction $\frac{2}{5}$ is equivalent to:
- .5
 - .04
 - .4
 - .05
6. $.6\overline{6}$. . . is equivalent to:
- $\frac{2}{3}$
 - $\frac{6}{10}$
 - $\frac{2}{7}$
 - $\frac{7}{10}$
7. To completely order the integers, +2, -2, 0, we could only have:
- $-2 < 0 < +2$
 - $-2 < +2 < 0$
 - $+2 < -2 < 0$
 - $0 < -2 < +2$
8. Which of the following is NOT a correct replacement for \square in $\frac{2}{5} \times \square = \frac{2}{5}$:
- $\frac{1}{1}$
 - 1
 - $\frac{2}{2}$
 - $\frac{1}{2}$

9. $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times N$. Which of the following should be a replacement for N:

A. $\frac{5}{4}$

B. $\frac{5}{3}$

C. $\frac{4}{5}$

D. $\frac{3}{5}$

10. If $\frac{16}{9} \times \frac{3}{2} = \frac{8}{3}$ then $\frac{8}{3} \div \frac{3}{2} = \frac{16}{9}$ or which of the following:

A. $\frac{8}{3} \div \frac{16}{9} = \frac{3}{2}$

B. $\frac{16}{9} \div \frac{3}{2} = \frac{8}{3}$

C. $\frac{8}{3} \times \frac{3}{2} = \frac{16}{9}$

D. $\frac{3}{2} \times \frac{8}{3} = \frac{16}{9}$

11. Consider the rational numbers 3 and 4.

A. $3\frac{3}{4}$ is the smallest rational number between them.

B. $3\frac{2}{4}$ is the smallest rational number between them.

C. $3\frac{1}{4}$ is the smallest rational number between them.

D. There is no smallest rational number between them.

12. Consider the sequence 5, 3, 1 . . .
The next integer should be:

A. 0
B. -1
C. -2
D. -3

13. Consider the number line:



The additive inverse of D is:

A. X
B. Zero
C. W
D. Z

14. An expression equivalent to 2 is:

A. $+\frac{1}{2}$
B. $+\frac{0}{2}$
C. $+\frac{2}{4}$
D. $+\frac{4}{2}$

15. $(2)^3$ means:

A. $2 \times 3 = 6$
B. $2 \times 2 \times 2 = 8$
C. $3 \times 2 = 6$
D. $3 \times 3 \times 3 = 27$

16. $\left(\frac{2}{3} \times \frac{4}{5}\right) \times \frac{7}{9} = \frac{2}{3} \times \left(\frac{4}{5} \times \frac{7}{9}\right)$ is an example of:
- The identity element of multiplication.
 - The associative property of multiplication.
 - The commutative property of multiplication.
 - The distributive property of multiplication.
17. $\frac{2}{3} \times \frac{3}{4} = \frac{3}{4} \times \frac{2}{3}$ is an example of:
- The commutative property of multiplication.
 - The associative property of multiplication.
 - The distributive property of multiplication.
 - The identity property of multiplication.
18. $\frac{2}{3} \times \left(\frac{3}{4} + \frac{5}{6}\right) =$
- $\left(\frac{2}{3} + \frac{3}{4}\right) \times \left(\frac{2}{3} + \frac{5}{6}\right)$
 - $\left(\frac{2}{3} \times \frac{3}{4}\right) + \frac{5}{6}$
 - $\left(\frac{2}{3} \times \frac{3}{4}\right) + \left(\frac{2}{3} \times \frac{5}{6}\right)$
 - $\left(\frac{2}{3} + \frac{3}{4}\right) \times \frac{5}{6}$
19. $(-6) + (-2) =$
- $(+6) + (+2)$
 - $(+2) + (-6)$
 - $(-2) + (-6)$
 - $(+6) + (-2)$

20. Which of the following is always true:

A. There is no rational number between

$$\frac{1}{3} \text{ and } \frac{1}{2} .$$

B. There is exactly one rational number between

$$\frac{1}{3} \text{ and } \frac{1}{2} .$$

C. There are exactly ten rational numbers

$$\text{between } \frac{1}{3} \text{ and } \frac{1}{2} .$$

D. There are an infinite number of rational

$$\text{numbers between } \frac{1}{3} \text{ and } \frac{1}{2} .$$

21. 3% is equal to the ratio:

A. $\frac{3}{10}$

B. .3

C. $\frac{3}{100}$

D. .30%

22. 40 is what percent of 60 may be written:

A. $\frac{40}{100} = \frac{N}{60}$

B. $\frac{40}{60} = \frac{N}{100}$

C. $\frac{100}{N} = \frac{40}{60}$

D. $\frac{60}{40} = \frac{N}{100}$

23. Which of the following would be used to convert $\frac{4}{5}$ to a decimal:

A. $\frac{5}{4} = \frac{N}{100}$

B. $\frac{4}{100} = \frac{5}{N}$

C. $\frac{4}{5} = \frac{100}{N}$

D. $\frac{4}{5} = \frac{N}{100}$

24. Find the correct replacement for N in the following:

$$\frac{.1}{.02} = \frac{N}{.004}$$

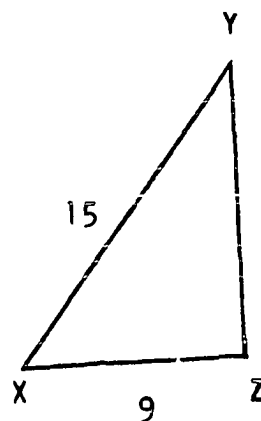
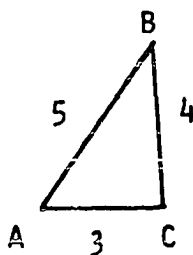
A. .02

B. .08

C. .0002

D. .00008

25. Triangles ABC and XYZ are similar. The correct length for \overline{YZ} is:



- A. 16
- B. 12
- C. 8
- D. 4

26. $\frac{1}{2} \div \frac{2}{3} =$

A. $\frac{4}{3}$

B. $\frac{3}{4}$

C. $\frac{3}{2}$

D. $\frac{2}{3}$

27. $\frac{3}{4} \times \frac{2}{3} =$

A. $\frac{5}{12}$

B. $\frac{1}{2}$

C. $\frac{5}{7}$

D. $\frac{6}{7}$

28. Consider the following division problem:

$$\begin{array}{r} \square 6 \text{ R } 14 \\ 93 \overline{) 8942} \\ \underline{837} \\ 572 \\ \underline{558} \\ 14 \end{array}$$

The missing number in the \square is:

A. 7

B. 8

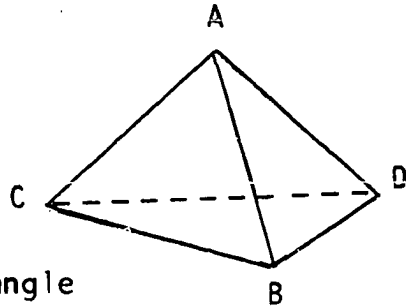
C. 9

D. 10

29. $(-15) + (-32) + (+8) =$

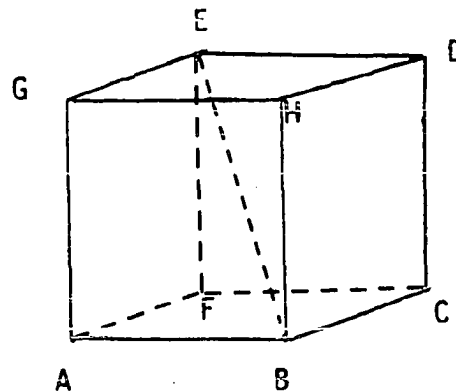
- A. +39
- B. +25
- C. -9
- D. -39

30. The 3-dimensional figure ABCD is called a(n):



- A. equilateral triangle
- B. equiangular prism
- C. tetrahedron
- D. icosahedron

31. Consider the cube:



Line segment BE is called a(n):

- A. edge
- B. face
- C. diagonal
- D. vertex

32. A scalene triangle may also be:

- A. equilateral
- B. equiangular
- C. isosceles
- D. right angled

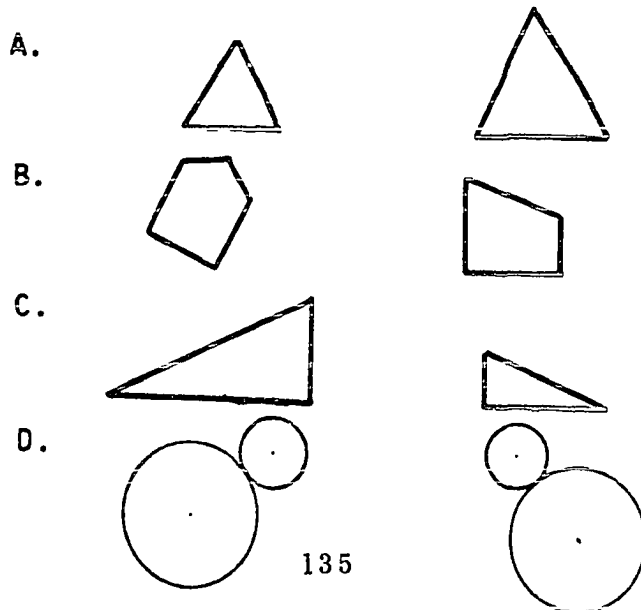
33. Which of the following is never true?

- A. A plane is a subset of a point.
- B. A point is a subset of a ray.
- C. A ray is a subset of a line.
- D. A line is a subset of a plane.

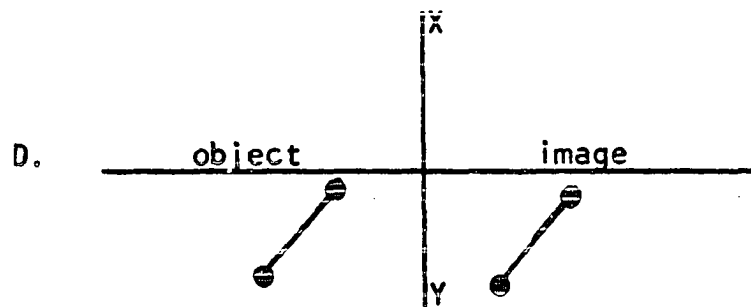
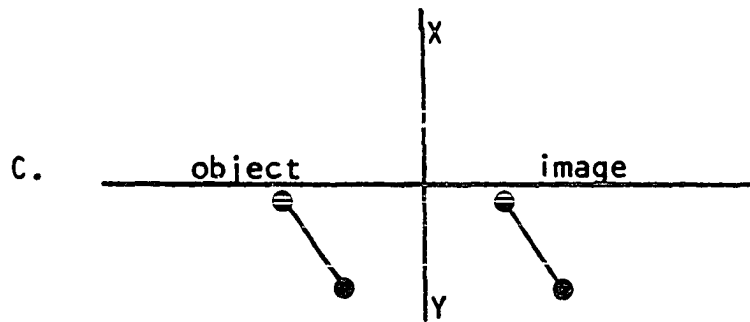
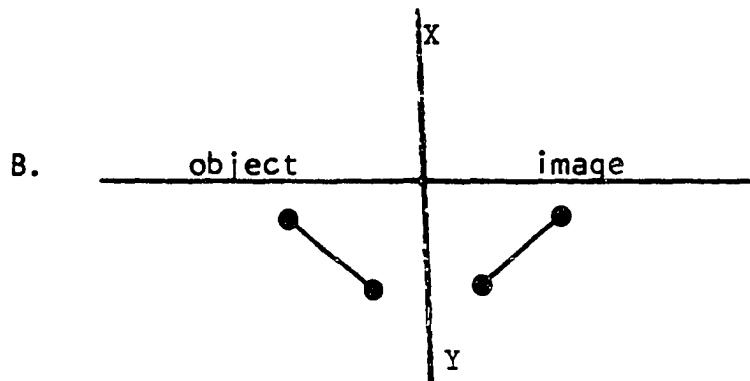
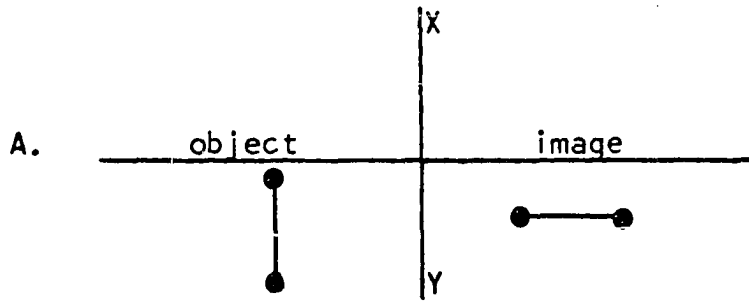
34. The circumference of a circle is about how many times the diameter:

- A. $1\frac{1}{7}$
- B. $2\frac{1}{7}$
- C. $3\frac{1}{7}$
- D. $4\frac{1}{7}$

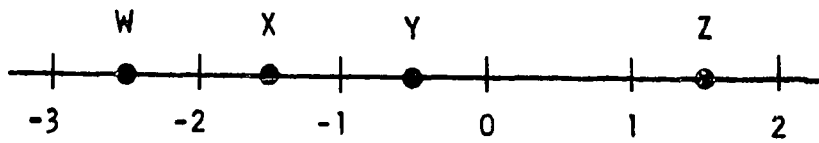
35. In which of the following are the figures congruent:



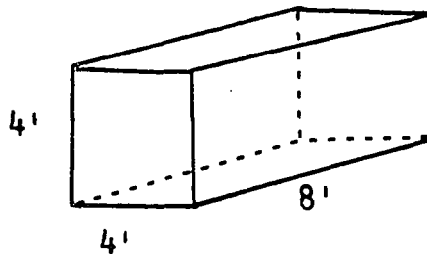
36. Which diagram illustrates that the image is a reflection of the object about the line XY:



37. Which point represents $-1\frac{1}{2}$:

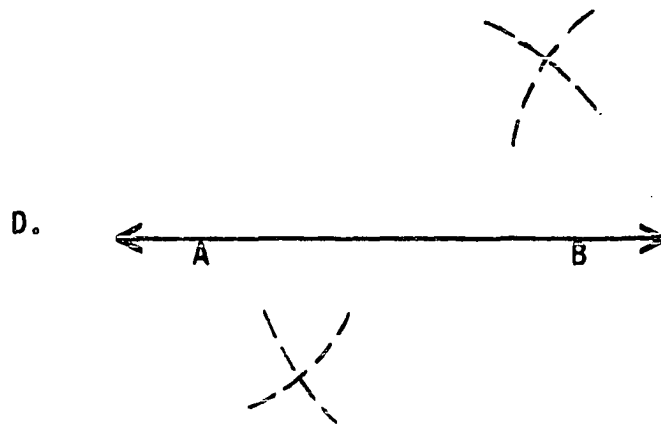
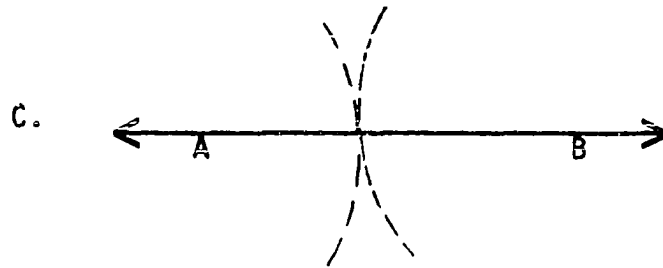
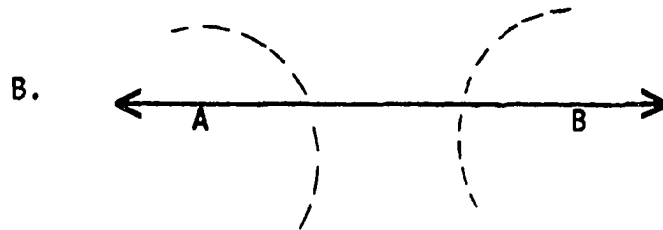
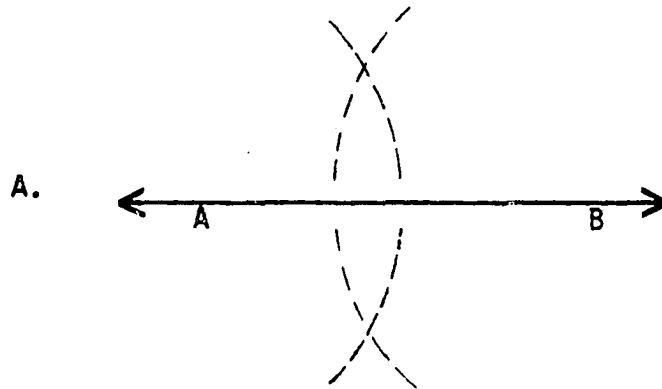


- A. W
 B. X
 C. Y
 D. Z
38. The number of cubic feet in the rectangular prism shown is:

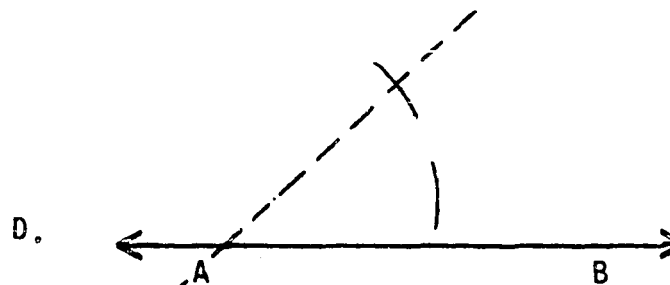
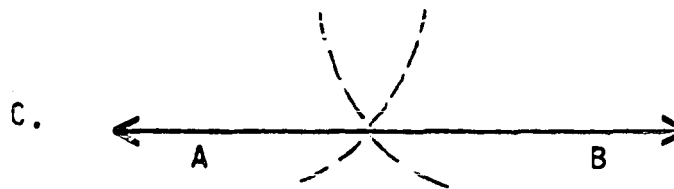
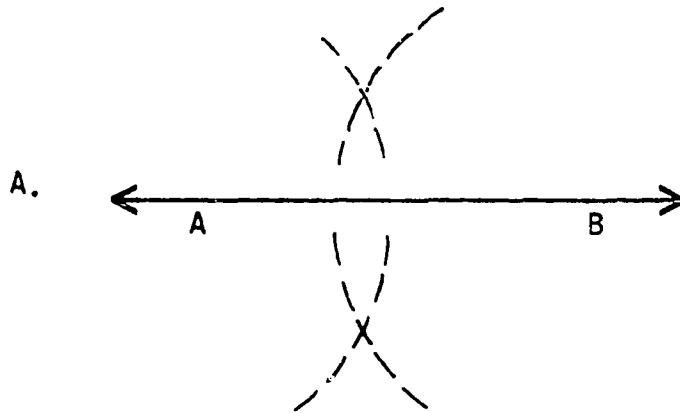


- A. 128
 B. 32
 C. 24
 D. 16
39. The formula $A = l \times w$ represents:
- A. The area of a circle
 B. The perimeter of a square
 C. The area of a rectangle
 D. The altitude of a triangle

40. You are given a line AB and are asked to construct a line perpendicular to it. Which of the following represents the correct method of beginning?

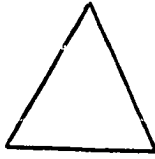


41. You are given a line AB and are asked to construct a line parallel to it. Which of the following represents the correct method of beginning?



42. Which of the following ³⁷⁵polygons has the largest perimeter?

A.



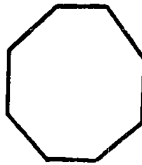
B.



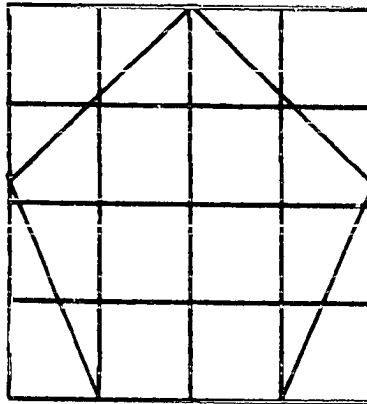
C.



D.

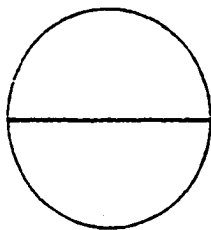


43. The best estimate of the area enclosed by the pentagon among the following is:

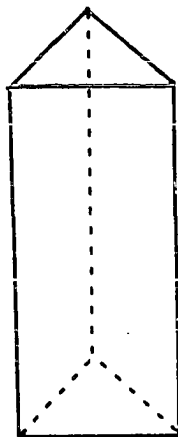


- A. 22 sq. units
- B. 16 sq. units
- C. 10 sq. units
- D. 6 sq. units

44. If $C = \pi d$ and $d = 14$ and assuming that $\pi = 3 \frac{1}{7}$ then the circumference of the circle is



- A. 22 in.
 B. 28 in.
 C. 32 in.
 D. 44 in.
45. How many centimeters wide is a 16 millimeter film?
- A. .16
 B. 1.6
 C. 16.0
 D. 160
46. If $V = 54$ cu. in. and $B = 9$ sq. in.
 Then $h =$:



- A. 27
 B. 18
 C. 6
 D. $4 \frac{1}{2}$

47. A rectangle 6.3 wide by 8.2 long has an area between:
- A. 6.1×8.1 and 6.2×8.2
 - B. 6.2×8.1 and 6.4×8.3
 - C. $6.2 + 8.1$ and $6.4 + 8.3$
 - D. $6.3 + 8.2$ and $6.4 + 8.3$
48. A 3 ft. stick casts a shadow of 4 ft., while a flagpole casts a shadow of 60 ft. at the same time. How high is the flagpole?
- A. 15 ft.
 - B. 20 ft.
 - C. 45 ft.
 - D. 80 ft.
49. If $.45 = \square \times .5$, then $\square =$
- A. .225
 - B. 2.25
 - C. .09
 - D. .9

SIXTH GRADE
ANSWER KEY

- | | | |
|-------|-------|-------|
| 1. B | 21. C | 41. D |
| 2. A | 22. B | 42. C |
| 3. A | 23. D | 43. C |
| 4. A | 24. A | 44. D |
| 5. C | 25. B | 45. B |
| 6. A | 26. B | 46. C |
| 7. A | 27. B | 47. B |
| 8. D | 28. C | 48. C |
| 9. A | 29. D | 49. D |
| 10. A | 30. C | |
| 11. D | 31. C | |
| 12. B | 32. D | |
| 13. D | 33. A | |
| 14. D | 34. C | |
| 15. B | 35. D | |
| 16. B | 36. B | |
| 17. A | 37. B | |
| 18. C | 38. A | |
| 19. C | 39. C | |
| 20. D | 40. A | |

Following is the coding of items in the 6th grade test as they relate to the behavioral objectives in Wisconsin K-6 Mathematics Guidelines. The first numeral in the code relates to grade level. The second numeral relates to horizontal strand number (1-15). The strands are found on the fold-out pages. The third numeral is the sub-objective as numbered in the Guideline.

SIXTH GRADE

<u>PROBLEM</u>	<u>CODE</u>	<u>PROBLEM</u>	<u>CODE</u>
1.....	6-01-1	26.....	6-06-1
2.....	6-02-1	27.....	6-06-1
3.....	6-02-2	28.....	6-06-2
4.....	6-02-3	29.....	6-06-3
5.....	6-02-4	30.....	6-07-1
6.....	6-02-4	31.....	6-07-2
7.....	6-03-1	32.....	6-08-1
8.....	6-04-1	33.....	6-08-2
9.....	6-04-2	34.....	6-08-3
10.....	6-04-3	35.....	6-10-1
11.....	6-04-4	36.....	6-09-1
12.....	6-04-5	37.....	6-12-3
13.....	6-04-6	38.....	6-14-1
14.....	6-04-7	39.....	6-14-4
15.....	6-04-8	40.....	6-13-1
16.....	6-04-9	41.....	6-13-2
17.....	6-04-9	42.....	6-14-2
18.....	6-4-10	43.....	6-14-3
19.....	6-4-11	44.....	6-14-5
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22.....	6-05-2	47.....	6-14-8
23.....	6-05-3	48.....	6-14-9
24.....	6-05-4	49.....	6-15-1
25.....	6-05-5		